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Original Research

Simulation Study of the Application of Hilbert Transform in Two-phase Flow Parameters Measurements using Gamma-ray Absorption

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Abstract

Measuring of parameters of two-phase flows usually needs the contactless measuring techniques to be used together with advanced methods of signal processing. One of these techniques, which are employed for many years in measurements of liquid-gas, liquid-solids and gas-solid particles flows is a method of gamma-ray densitometry. Frequently in such measurements the mutually delayed stochastic signals are received from the scintillation detectors. For the time delay estimation the well-known cross-correlation method is usually used due to the random nature of the signals and presence of disturbances. This paper describes a proposition of use of Hilbert Transform to time delay estimation in radioisotope measurements of two-phase flow. It presents results of simulation study of the modified cross-correlation method, in which the Hilbert Transform of one measured signal is used. The simulations have been carried out for models of stochastic signals, corresponding to signals received in investigations of liquid-gas flow through horizontal pipeline, carried out with use of gamma-ray absorption technique. It has been stated that the described method provides better metrological properties than classical cross-correlation.

Keywords: two-phase flow, gamma ray absorption, random signals, time delay estimation, Hilbert Transform, cross-correlation

1. Introduction

Two-phase flows occur in nature and industry, e.g., in the transportation of mixtures as liquid-gas, or liquid-solids in pipelines. Measuring the parameters of these components is difficult and frequently requires the use of non-invasive methods [1]. One of these techniques, which are employed for many years, is a gamma-ray absorption method [2-12]. This method is relatively simple in principle, but need to meet restrictive safety requirements for protecting the personnel and the environment from the effects of the ionizing radiation.

Frequently in radioisotope measurements the mutually delayed stochastic signals are provided by the scintillation probes. The time delay of these signals is used to determine the velocity of the dispersed phase or other flow parameters. Processing of measured signals, usually disturbed by noises, needs the use of signal conditioning and statistical processing methods in time or frequency domain. The classical methods of time delay estimation (TDE) used for stationary signals include the cross-correlation function (CCF) or the phase of cross-spectral density [5, 10, 13-19]. A less popular method of TDE includes, among others, the correlation analysis using Hilbert Transform [14, 20-23], differential methods [24] and methods based on the conditional averaging of the signals [25-26]. Due to the fact that usefulness of the last three methods in the two-phase flow measurements using radioisotopes is not known in detail, they can be, in the first stage, examined by means of simulation.

In the first part of this paper the models of signals have been described, which is used for TDE of the random signals. Then the cross-correlation method is described together with the possibility of use of Hilbert Transform in the correlation measurements. The further part of this paper presents examples of results of simulation studies of the method, in which the Hilbert Transform of one measurement signal has been used for cross-correlation analysis. The studies have been carried out for computer-generated models of mutually delayed stochastic signals. Parameters of these models have been selected in a way ensuring a proper correspondence with the real signals obtained in measurements of water-air flow in a horizontal pipeline, performed with the use of gamma-ray absorption. The received results for time delays and their standard uncertainties have then been compared with the corresponding results obtained by means of classical cross-correlation.

This paper is a revised and extended version of the conference work [27].

2. Models of signals

TDE problems deal with signals $x(t)$ and $y(t)$ received from two sensors, which can be expressed by the formulas [14, 28]:

$$x(t) = s(t) + m(t) \tag{1}$$

$$y(t) = c \cdot s(t - \tau_0) + n(t) \tag{2}$$

where: $s(t)$ is a stationary random signal with normal $N(0, \sigma_s)$ distribution of probability, frequency band B and one-sided power spectral density:

$$G_{ss}(f) = \begin{cases} K & f \le B \\ 0 & f > B \end{cases} \tag{3}$$

c is a constant coefficient; τ_0 is a transportation time delay; $m(t)$, $n(t)$ are white noises with Gaussian distributions $N(0,\sigma_m)$, $N(0,\sigma_n)$, not correlated with $s(t)$ signal and not cross-correlated. The autocorrelation function of the $s(t)$ signal can be expressed as:

$$R_{ss}(\tau) = KB\left(\frac{\sin 2\pi B\tau}{2\pi B\tau}\right) \tag{4}$$

The following can be stated under the mentioned above assumptions concerning the models of signals (1) and (2):

$$\sigma_x^2 = \sigma_s^2 + \sigma_m^2 \tag{5}$$

$$\sigma_y^2 = c^2\sigma_s^2 + \sigma_n^2 \tag{6}$$

where σ_x and σ_y are the standard deviations of $x(t)$ and $y(t)$ signals respectively.

The signal to noise ratio (SNR) for signals (1) and (2) can be defined correspondingly as: $SNR_x = (\sigma_s/\sigma_m)^2$ for $x(t)$ and $SNR_y = (\sigma_s/\sigma_n)^2$ for $y(t)$. Depending on the presence of noise in one or both measurement channels, three models of signals can be considered [25], but in practice more frequently only two cases are considered:

- model I: $\sigma_m = 0$, $\sigma_n \neq 0 = \sigma_z$ and $SNR_y = SNR$; then:

$$y(t) = c \cdot s(t-\tau_0) + z(t) = c \cdot x(t-\tau_0) + z(t) \tag{7}$$

$$SNR = (\sigma_s/\sigma_z)^2 = (\sigma_x/\sigma_z)^2 \tag{8}$$

where $z(t)$ is white noise with Gaussian $N(0,\sigma_z)$ distribution;

- model II: $\sigma_m = \sigma_n \neq 0 = \sigma_z$ and $SNR_x = SNR_y = SNR$; then:

$$x(t) = s(t) + z_1(t) \tag{9}$$

$$y(t) = c \cdot s(t-\tau_0) + z_2(t) \tag{10}$$

$$SNR = (\sigma_s/\sigma_z)^2 \tag{11}$$

The disturbing $z_1(t)$ and $z_2(t)$ white noise signals in the two channels have identical normal distributions $N(0,\sigma_z)$, but these are distinct, not cross-correlated realizations.

3. Cross-correlation principle of TDE

A cross-correlation function of $x(t)$ and $y(t)$ signals is equal to [14]:

$$R_{xy}(\tau) = E[(x(t)y(t+\tau)] = c\,R_{ss}(\tau-\tau_0) \tag{12}$$

where: $E[\cdot]$ - expected value, τ - time delay, $R_{ss}(\cdot)$ - autocorrelation function of signal $s(t)$.
The CCF reaches its max value for $\tau = \tau_0$, so the transportation time delay can be determined as the argument of the maximum of this function (Fig. 1).

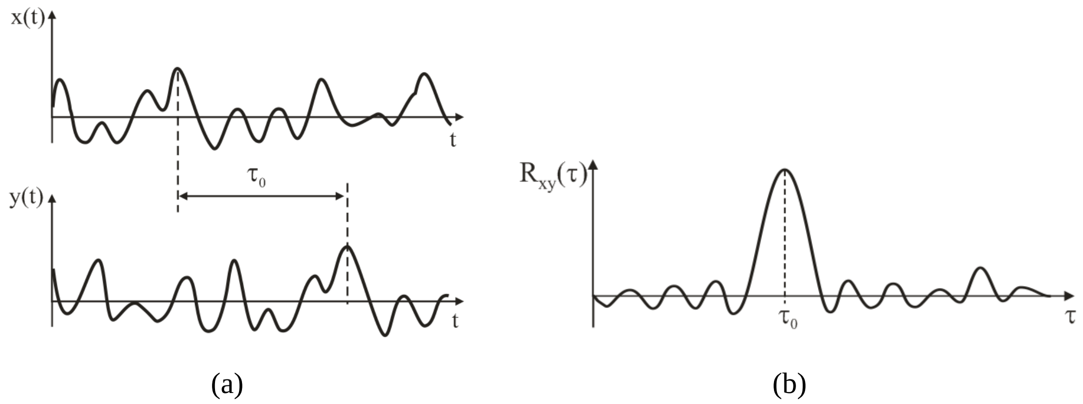


Fig. 1. The concept of TDE τ_0 from the cross-correlation function: a) the waveforms $x(t)$ and $y(t)$; b) the CCF $R_{xy}(\tau)$.

The normalised CCF for $\tau = \tau_0$ can be written as the following expression [13, 14]:

$$\rho_{xy}(\tau_0) = \frac{R_{xy}(\tau_0)}{\sqrt{R_{xx}(0)R_{yy}(0)}} = \frac{cR_{ss}(0)}{\sigma_x\sigma_y} = \frac{c\sigma_s^2}{\sigma_x\sigma_y} \tag{13}$$

where $R_{xx}(\cdot)$ and $R_{yy}(\cdot)$ are the autocorrelation functions of signals $x(t)$ and $y(t)$.
When we substitute in (13) the formulas (5) and (6), as well as $c = 1$, then the following $\rho_{xy}(\tau_0) = f(SNR)$ expressions will be obtained:

● model I - for (7) signal:

$$\rho_{xy}(\tau_0) = \left[1+\left(\frac{\sigma_z}{\sigma_s}\right)^2\right]^{-1/2} = \left[1+\frac{1}{SNR}\right]^{-1/2} \tag{14}$$

● model II - for (9) and (10) signals:

$$\rho_{xy}(\tau_0) = \left[\left(1+\frac{\sigma_z^2}{\sigma_s^2}\right)^2\right]^{-1/2} = \left[1+\frac{1}{SNR}\right]^{-1} \tag{15}$$

The equations (14) and (15) can be useful in determination of *SNR* on the base of measured normalised CCF.

The discrete estimator for the cross-correlation function can be expressed as the following formula [17, 28]:

$$\hat{R}_{xy}(l) = \frac{1}{N}\sum_{n=0}^{N-1} x(n)y(n+l) \tag{16}$$

where N is the number of discrete values of $x(n)$ and $y(n)$ signals sampled with Δt time interval ($n = t/\Delta t$, $l = \tau/\Delta t$).

4. Application of Hilbert Transform to cross-correlation TDE of the random signals

Hilbert Transform (HT) of real signal $x(t)$ gives a real-valued signal $\tilde{x}(t)$ in accordance with the definition [14, 20]:

$$\tilde{x}(t) = H[x(t)] = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(u)}{(t-u)} du \tag{17}$$

Signal $\tilde{x}(t)$ is then a convolution $x(t) * (1/\pi t)$. The Hilbert Transform can be used for realisation of analytic signal

$$\underline{x(t)} = x(t) + jH[x(t)] = x(t) + j\tilde{x}(t) \tag{18}$$

A modulus of the analytic signal:

$$\left|\underline{x(t)}\right| = \sqrt{x^2(t) + \tilde{x}^2(t)} \tag{19}$$

is called the envelope of the signal $x(t)$.

In the literature one can find a number of possibilities of HT and analytic signal use in cross-correlation measurements of time delay of random signals [14, 20-23, 29-32]. Papers [20, 21] contains a proposition of use $\tilde{y}(t)$ (HT of the signal $y(t)$) for calculation of cross-correlation function instead of the signal $y(t)$ itself. The cross-correlation function with Hilbert Transform (CCFHT) obtained in such a way, expressed by a formula:

$$R_{x\tilde{y}}(\tau) = \tilde{R}_{xy}(\tau) = E[(x(t)\tilde{y}(t+\tau)] \tag{20}$$

assumes zero value for $\tau = \tau_0$. Calculation of the time coordinate of the maximum of CCF (12) can then be replaced by looking for τ values where the CCFHT function is equal to zero, the realisation of which is simpler. Examples of graphical representations of CCF and CCFHT functions are presented in Fig. 2.

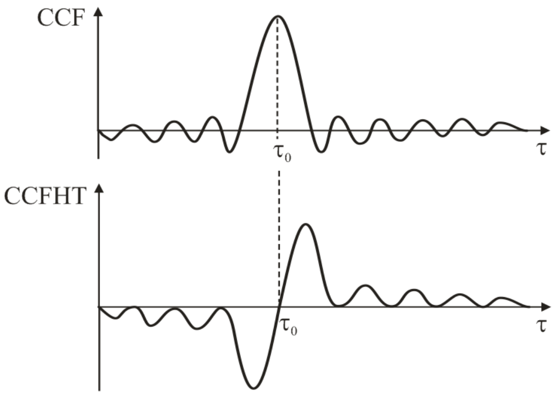


Fig. 2. Examples of graphical representations of CCF and CCFHT.

The discrete estimator of the CCFHT can be expressed as:

$$\hat{\tilde{R}}_{xy}(l)=\frac{1}{N}\sum_{n=0}^{N-1}x(n)\tilde{y}(n+l) \tag{21}$$

The analyses presented in [20, 23, 31] show that the standard deviation of time delay τ_0 by the CCFHT is lower comparing with CCF one and envelopes in case of non-correlated samples of low-pass random signals.

5. Gamma-ray absorption in two-phase flow measurements

Figure 3 presents the principle of measuring the flow velocity of a gas phase transported by liquid in a horizontal pipeline using single-beam gamma densitometry.

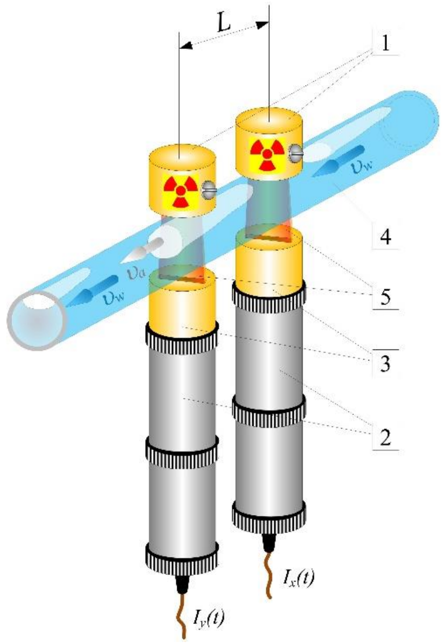


Fig. 3. The principle of the gamma absorption measurement of the two-phase flow: 1 – gamma ray sources with collimators, 2 – scintillation probes, 3 – detectors' collimators, 4 – pipeline, 5 – the main beam of gamma rays, υ_a – velocity of air, υ_W – velocity of water.

Beams of gamma rays emitted by the sources and formed by collimators are partially absorbed by the flowing medium [2, 5, 7-10]. Electronic pulses $I_x(t)$ and $I_y(t)$, received on the outputs of scintillation probes, situated at the L distance on the opposite side of the pipeline, are counted down within Δt sampling intervals, and giving the mutually delayed discrete stochastic signals $x(n)$ and $y(n)$. These signals describe the instantaneous states of the flowing medium in the examined sections.

Based on the measured τ_0 time delay of these signals, the average velocity of a gas phase

$$\upsilon_p = L / \tau_0 \tag{22}$$

can be calculated as well as other flow parameters [5, 6, 10, 13, 28]. Signals from the detectors, after centering and filtration, are ergodic and can be analyzed using statistical methods in time or frequency domain.

Figure 4 presents the normalised cross-correlation functions obtained for the signals received in the selected experiments BUB 005, BUB 006, and BUB 010 for water-air flow through the pipeline of 30 mm inner diameter.

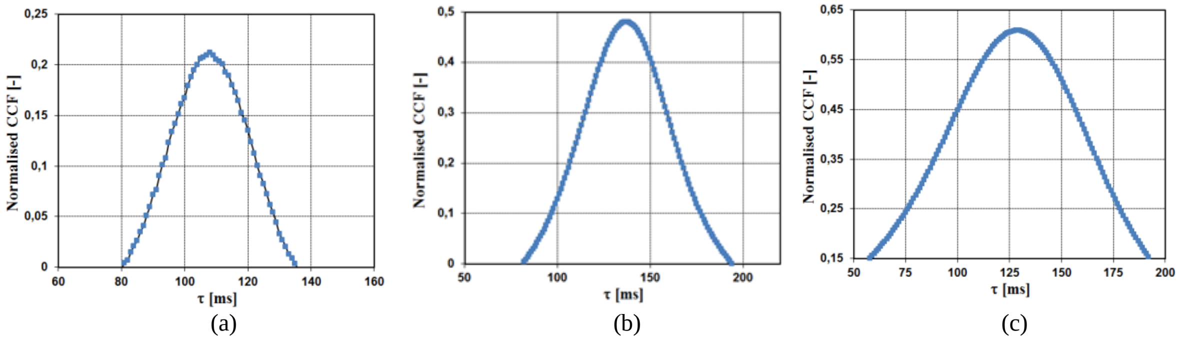


Fig. 4. Graphical representation of normalised CCF obtained in the experiment (a) BUB 005, (b) BUB 006, and (c) BUB 010.

Acquisition parameters were as follows: $N = 300{,}000$, $\Delta t = 1$ ms. A closed ^{241}Am gamma ray source of 59.5 keV energy has been used in the experiment mentioned above together with scintillation detectors based on NaI(Tl) crystals. The laboratory stand and geometry of the used gamma absorption set are described in detail in papers [7, 8, 10, 32].

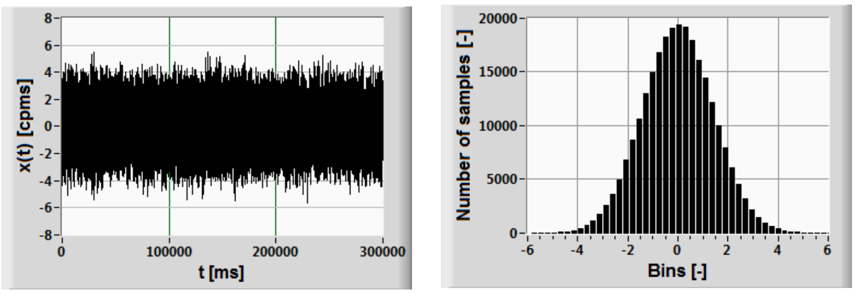
6. Results of simulation studies

6.1. Modelling of signals

Signals received in flow measurements from the gamma ray detectors contain not only statistical information about the examined flow, but also disturbing signals caused by the gamma radiation background, equipment noise and fluctuations in decay rate. Such signals can be modelled with use of the models described in Section 2, with properly selected parameters. In the study reported here the model II, defined by formulas (9) and (10), has been used. The $s(n)$ signal has been formed of the white noise with use of a low-pass filter with parameters and *SNR* selected in a way ensuring the shape and amplitude of the normalised cross-correlations similar to the CCF functions shown in Fig. 4. The $z_1(n)$ and $z_2(n)$ disturbing signals have been Gaussian white noises with $N(0, \sigma_z)$ distributions, not cross-correlated and not correlated with the useful signals. Figure 5 shows the exemplary time waveform and histogram of the modeled signal. Figure 6 presents the normalised cross-correlation functions obtained by the modeling with parameters given in Table 1.

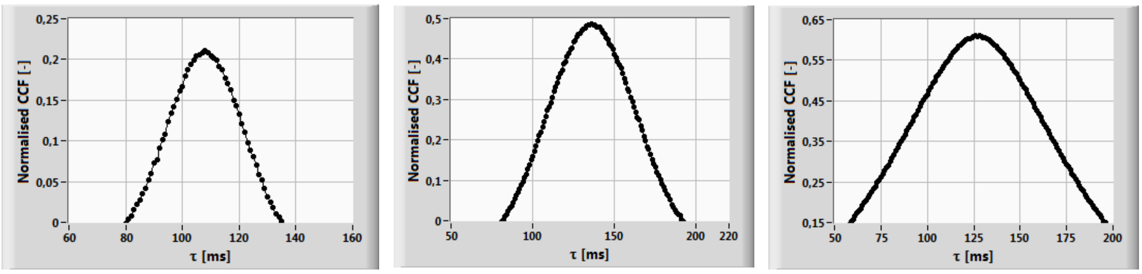
Table 1. Parameters of simulations.

Modelled experiment	τ_0 **[ms]**	***N*** **[-]**	***SNR*** **[-]**	**LP filter relative cut-off frequency**
BUB 005	108	300,000	0.27	0.018
BUB 006	136	300,000	0.96	0.009
BUB 010	128	300,000	1.56	0.005



(a) (b)

Fig. 5. The exemplary waveform (a) and histogram (b) of the modeled signal for BUB 006 experiment.



(a) (b) (c)

Fig. 6. Normalised CCF functions obtained by modelling for (a) BUB 005, (b) BUB 006, and (c) BUB 010 experiments.

For models of signals with the given parameters the functions CCF (16) and CCFHT (21) has been determined. The resulting graphical representations for the BUB 006 are presented in Fig. 7.

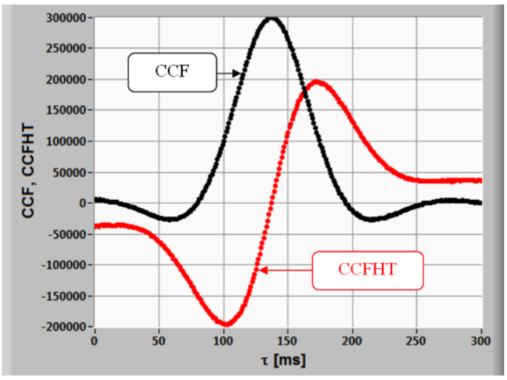


Fig. 7. Graphs of CCF and CCFHT obtained through modelling.

Bendat in work [20] presents expressions describing the CCF and CCFHT estimation errors. In the paper [23] these errors have been compared for models of signals (1), (2), while in the article [31] the results of comparative simulation studies have been described.

The formulas given in the papers mentioned above can be used in case of non-correlated pairs of samples of signals $x(n)$ and $y(n)$.

The samples of signals obtained in gamma-ray absorption measurements described in Section 6.1, are correlated and this is why the procedures presented below have been proposed for the purpose of determination of the standard uncertainty of time delay.

6.2. Estimation of the standard uncertainty of transportation time delay

For determination of time delay based on CCFHT we have been used the approximation of m points of this function with a straight line described as:

$$y = a_0 + a_1\tau \tag{23}$$

The graphs of CCFHT function and the straight line obtained for m = 30 points are shown in Fig. 8.

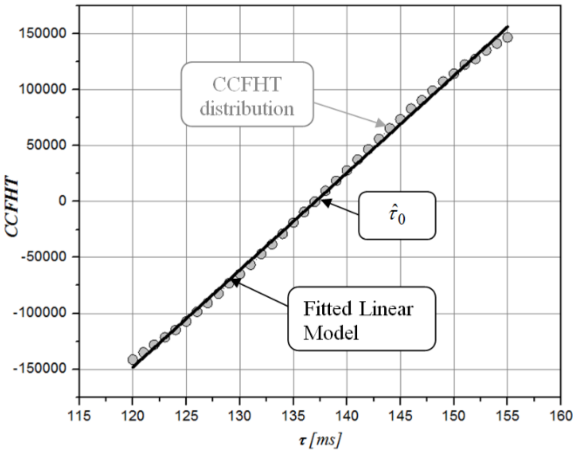


Fig. 8. Approximation of function CCFHT.

For $\tau = \tau_0$ it is true that $y = 0$ and from the equation (23) we can obtain:

$$\hat{\tau}_0 = -\frac{a_0}{a_1} \tag{24}$$

The standard uncertainty of time delay $u(\hat{\tau}_0)$ can be calculated from the formula [32]:

$$u[\hat{\tau}_0] = \left[\frac{\sum_{i=1}^{m} \left(a_0 + a_1 \tau_i - \tilde{R}_{xyi} \right)^2}{m(m-2)a_1^2} \right]^{1/2} \tag{25}$$

In the case described here we have obtained a_1 = 8930,94 and a_0 = -1222469,94. The calculated values of time delay and $u(\hat{\tau}_0)$ are listed in Table 2.

Table 2. Results of TDE for modelled experiments.

Method	$\hat{\tau}_0$ **[ms]**	$u(\hat{\tau}_0)$ **[ms]**	$u(\hat{\tau}_0)/u(\hat{\tau}_0)_{CCF}$ **[-]**	**Modelled experiment**
CCF	108.09	1.76	1.00	BUB005 k=61, m=19
CCFHT	108.07	0.07	0.04	
CCF	136.87	2.61	1.00	BUB006 k=115, m=30
CCFHT	136.88	0.06	0.03	
CCF	127.06	3.16	1.00	BUB010 k=112, m=21
CCFHT	127.33	0.07	0.03	

Determination of the transportation time delay on the base of CCF consists in looking for the argument at the function's maximum. For this purpose a parabola approximation can be used, based on discrete points of cross-correlation in the neighborhood of the maximum, or approximation with Gaussian distribution [5, 17, 33]. In this study we have used approximation of the selected part of CCF with use of the following Gauss function:

$$p(\tau) = p_0 + \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(\tau - \hat{\tau}_0)^2}{2\sigma^2}\right) \quad (26)$$

where p_0 - normalization level of the Gauss function, σ - standard deviation of the fitted distribution. In such case the $\hat{\tau}_0$ transportation time estimator is determined by means of the first moment of the fitted normal distribution [5, 34], while the $u(\hat{\tau}_0)$ standard uncertainty is equal to the standard deviation of the mean value [35]:

$$u(\hat{\tau}_0) = \frac{\sigma}{\sqrt{k}} \quad (27)$$

where k is a number of samples used in the approximation procedure. The obtained values of $\hat{\tau}_0$ time delay and $u(\hat{\tau}_0)$ are listed in Table 2.

Figure 9 presents a result of CCF approximation obtained for k = 115.

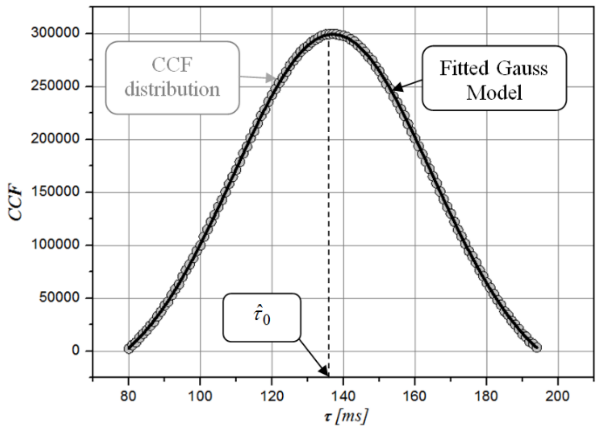


Fig. 9. Results of CCF approximation.

7. Conclusion

The paper describes the possibilities of Hilbert Transform applying in the cross-correlation measurements of time delay of random signals. It contains a proposition of CCFHT use in two-phase flow investigations performed on the base of gamma-ray densitometry. In this method the Hilbert Transform of one signal is used for the purpose of cross-correlation analysis. We present here the examples of results of CCFHT simulation study and, for comparison, the classic CCF method as well. The simulations have been performed for computer-generated models of stochastic signals, corresponding to signals received from scintillation detectors, after preprocessing, in measurements of water-air flow velocity in horizontal pipeline. For each function we have determined the values of time delay and its standard uncertainty, based on the linear approximation for CCFHT and Gaussian approximation for CCF. It has been stated that CCFHT method guaranties lower values of standard uncertainty of time delay than the CCF one. In the presented case the CCFHT uncertainty has been nearly 25 times lower than that of CCF. The CCFHT method presented in this paper can also be applied for liquid-solid particles flow analysis in a vertical pipeline [36].

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