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On Some Classes of Block Repetition Codes with Covering Radius of the Codes $\mathbb{Z}_{3^2}$

Chella Pandian P.

Abstract: In this paper, to obtain the bounds for some classes of repetition codes with covering radius by using various weight and also the same size and different size of length in repetition codes over a finite ring $\mathbb{Z}_{3^2}$.

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1. Introduction

In coding theory for the last five decades, many researchers has been attraction in codes over finite rings and the special types of the rings $\mathbb{Z}_{2n}$, where $2n$ is the ring of integers modulo.

The authors was discovered the best well known non-linear binary codes can be constructed by cyclic codes and gray map over a finite ring $\mathbb{Z}_4$ in [19] and many research articles has indicated codes over a finite ring $\mathbb{Z}_4$ received much attention [1, 5–9]. Coding theory, the covering radius is one of the important parameter to find the maximum error-correcting capability of codes. In Binary code, [3, 4, 13–15], the covering radius of codes are studied for linear and non-linear codes can be received from codes over a finite ring $\mathbb{Z}_4$ via the gray map. In [10–12], the author to find lower bound and upper bound of covering radius in a suitable of different types repetition codes by using some finite rings with respect to various weight.

In this paper, to determine the covering radius of some attraction classes of repetition codes over a finite commutative ring $\mathbb{Z}_{3^2}$ of interger modulo 3^2 by using to different weight(distance).

2. Preliminaries

Let $\mathbb{Z}_{3^2}$ be a finite set with nine elements $\{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ with two operation $\oplus_{3^2}, \odot_{3^2}$ is said to be a finite commutative ring. It is denoted by $(\mathbb{Z}_{3^2}, \oplus_{3^2}, \odot_{3^2}) = \mathbb{Z}$ with a characteristic 3^2. Let $C \subseteq \mathbb{Z}$, then C is say that a *code*. A code C is called the *linear code*, if the ring $\mathbb{Z}$ is an $\mathbb{Z}$-submodule of $\mathbb{Z}^l$, where l is the length of a code(that is, $C = (11111)$, $l(C) = 5$, $C_1 = (3333)$, $l(C_1) = 4$). The elements of C is called a *codeword* of C.

A Gray Map $h : \mathbb{Z}_{3^2} \to (\mathbb{Z}_3 \times \mathbb{Z}_3)$ is defined by

$$h(0) = 00, h(1) = 01, h(2) = 02, h(3) = 10, h(4) = 11, h(5) = 12,$$

$$h(6) = 20, h(7) = 21, h(8) = 22,$$

then the Gray map $h_1 : \mathbb{Z}_{3^2}^l \to (\mathbb{Z}_3 \times \mathbb{Z}_3)^l$ is define $h_1(y) = (h(y_1), h(y_2), \cdots, h(y_n))$, where $y = (y_1, y_2, \cdots, y_n)$ in [17].

Let $y \in \mathbb{Z}^l$ be a codeword of code, that is $y = (y_1, y_2, \cdots, y_n)$ and in [20], the Lee weight of y as given

$$w_L(y) = \begin{cases} 0 & \text{if} \quad y = 0 \\ 1 & \text{if} \quad y = 1, 8 \\ 2 & \text{if} \quad y = 2, 7 \\ 3 & \text{if} \quad y = 3, 4, 5, 6. \end{cases}$$

Let $y_i \in \mathbb{Z}$ be the codeword of Lee weight of y_i is defined as $\sum_i w_L(y_i)_{,i=0 \text{ to } 8}$. If $c_1, c_2 \in C$, be any two distinct codewords of Lee distance is defined as $d_L(C) = \{d_L(c_1, c_2) | c_1 - c_2 \neq 0 \text{ and } c_1, c_2 \in C\}$. The minimum Lee weight of C is $d_L(C) = \min\{d_L(c_1, c_2) | c_1 - c_2 \neq 0 \text{ and } c_1, c_2 \in C\}$. In C is a *linear code* C, thus $d_L(C) = \min\{w_L(c) | c \neq 0 \in C\}$. Therefore, $d_L(c_1, c_2) = w_L(c_1 - c_2)$. If C is a linear code of length l over $\mathbb{Z}$ with the number of codewords W and the minimum Lee distance d_L, is said to be an (l, W, d_L) code in $\mathbb{Z}$. In C is a linear code of length l over $\mathbb{Z}$, therefore the Lee distance between z and C is defined as $d_L(z, C) = \min\{d_L(z, c) | \forall c \in C\}$, for any $z \in \mathbb{Z}^l$.

The Chinese Euclidean weight of x is

$$w_{CE}(y) = \begin{cases} 0 & \text{if} \quad y = 0 \\ 1 & \text{if} \quad y = 1, 8 \\ 2 & \text{if} \quad y = 2, 7 \\ 3 & \text{if} \quad y = 3, 6 \\ 4 & \text{if} \quad y = 4, 5 \end{cases}$$

in [18], where $y = (y_1, y_2, \cdots, y_n)$ be a codeword of code over $\mathbb{Z}^l$.

The parameters of Chinese Euclidean weight code is an (l, W, d_{CE}). In Chinese Euclidean distance(weight), let $c_1, c_2 \in \mathbb{Z}^l$ be any two different codewords is defined as $d_{CE}(c_1, c_2) = wt_{CE}(c_1 - c_2)$. Let C be a linear code of length l over $\mathbb{Z}$. Then $d_{CE}(z, C) = \min\{d_{CE}(z, c) | \forall c \in C\}$, for any $z \in \mathbb{Z}^l$.

In Gray weight, let $y \in \mathbb{Z}^l$ be a codeword of code, is define as

$$w_G(y) = \begin{cases} 0 & \text{if} \quad y = 0 \\ 1 & \text{if} \quad y = 1, 2, 3 \text{ and } 6 \\ 2 & \text{if} \quad \text{otherwise} \end{cases}$$

in [17].

In C is a linear code with Gray weight(distance), is an (l, W, d_G) code. Define, $d_G(c_1, c_2) = wt_G(c_1 - c_2)$, where $c_1, c_2 \in \mathbb{Z}^l$ and $d_G(z, C) = \min\{d_G(z, c) | \forall c \in C\}$, for any $z \in \mathbb{Z}^l$.

In [2], Let $y \in \mathbb{Z}^l$. The Bachoc weight of x is defined as

$$w_B(y) = \begin{cases} 0 & \text{if} \quad y = 0 \\ 1 & \text{if} \quad y = 1, 2, 4, 5, 7, 8, \\ 3 & \text{if} \quad y = 3, 6. \end{cases}$$

In C is a linear code with Bachoc weight(distance) is an (l, W, d_B) code. Define, $d_B(c_1, c_2) = wt_B(c_1 - c_2)$, where $c_1, c_2 \in \mathbb{Z}^n$ and $d_B(z, C) = \min\{d_B(z, c) | \forall c \in C\}$, for any $z \in \mathbb{Z}^n$.

Example 2.1. *Let* $y = 1\ 3\ 4\ 7\ 2 \in \mathbb{Z}^5$. *Then,*

$$w_L(y) = w_L(1) + w_L(3) + w_L(4) + w_L(7) + w_L(2) = 11,$$

$$w_{CE}(y) = w_{CE}(1) + w_{CE}(3) + w_{CE}(4) + w_{CE}(7) + w_{CE}(2) = 12,$$

$$w_G(y) = w_G(1) + w_G(3) + w_G(4) + w_G(7) + w_G(2) = 8 \textit{ and}$$

$$w_B(y) = w_B(1) + w_B(3) + w_B(4) + w_B(7) + w_B(2) = 10.$$

3. Repetition code with Covering radius of code in $\mathbb{Z}$

Let d be the distance of a code C in $\mathbb{Z}$ with respect to different distance(weight), such as Lee weight, Chinese Euclidean weight, Gray weight and Bachoc weight. The *covering radius* of a code C is

$$R_d(C) = \max_{w \in \mathbb{Z}^n} \left\{ \min_{c \in C} \{d(w, c)\} \right\},$$

where C is a code and $R_d(C)$ is a covering radius of the code C with distance d.

In $F_q = \{0, 1, \gamma_2, \cdots, \gamma_{q-1}\}$ is a finite field. Let C be a q-ary *repetition code* C over F_q. That is $C = \{\bar{\gamma} = (\gamma\gamma \cdots \gamma) | \gamma \in F_q\}$ and the repetition code C is an $[l, 1, l]$ code. Therefore, the covering radius of the code C is $\lfloor \frac{l(q-1)}{q} \rfloor$ by using in [16].

Let C be a block repetition code of size l, the parameter of C is an $[l(q-1), 1, l(q-1)]$ be a generated by $G = [\overbrace{11\cdots 1}^{l}\overbrace{\gamma_2\gamma_2\cdots\gamma_2}^{l}\cdots\overbrace{\gamma_{q-1}\gamma_{q-1}\cdots\gamma_{q-1}}^{l}]$. In [16], thus the covering radius of the code C is $\lfloor\frac{l(q-1)^2}{q}\rfloor$, since it will be equivalent to a repetition code of length $(q-1)l$.

A code $C \subseteq \mathbb{Z}$ is also linear code and is called the Generator matrix(G), if the basis elements in a row of matrix.

In repetition code over $\mathbb{Z}$, there are two type of repetition codes of length l viz.

1. Type A-(A Generator matrix(G_A) with unit element in $\mathbb{Z}$ and its generated by the code C_A)
2. Type B-(A Generator matrix(G_B) with zero divisor element in $\mathbb{Z}$ and its generated by the code C_B)

Type A (G_A) $\to$	$[\overbrace{11\cdots 1}^{l}], [\overbrace{22\cdots 2}^{l}], [\overbrace{44\cdots 4}^{l}]$, $[\overbrace{55\cdots 5}^{l}], [\overbrace{77\cdots 7}^{l}], [\overbrace{88\cdots 8}^{l}]$	$[l, k=1, d_i=l]$, $i = \{L, CE, G, B\}$
Type B (G_B) $\to$	$[\overbrace{33\cdots 3}^{l}], [\overbrace{66\cdots 6}^{l}]$, $[\overbrace{36\ 36\cdots 36}^{l}], [\overbrace{63\ 63\cdots 63}^{l}]$	$(l, W=3, d_j=3l)$, $j = \{L, CE, G, B\}$

Theorem 3.1.

- $R_L(C_A) = 2l$,
- $R_L(C_B) = 2l$, *here* $R_L(C_{A(B)})$ *is a covering radius of codes* $C_{A(B)}$ *for generator matrix* $G_{A(B)}$ *by using Lee weight and* l *is a length of code in Type A and Type B.*

Proof. Let $y \in \mathbb{Z}^l$ by ϱ_0 times $0'$s, ϱ_1 times $1'$s, ϱ_2 times $2'$s, ϱ_3 times $3'$s, ϱ_4 times $4'$s, ϱ_5 times $5'$s, ϱ_6 times $6'$s, ϱ_7 times $7'$s, ϱ_8 times $8'$s in y and $\sum_i \varrho_i = l$ and the code $c_i \in \{\gamma(C_A)|\gamma \in \mathbb{Z}^l\}$, where $i = 0$ to 8. Then

$$\begin{aligned} d_L(y, c_0) &= wt_L(y - 00\cdots 0) \\ &= 0\varrho_0 + 1\varrho_1 + 2\varrho_2 + 3\varrho_3 + 4\varrho_4 + 5\varrho_5 + 6\varrho_6 + 7\varrho_7 + 8\varrho_8 \\ &= \varrho_1 + 2\varrho_2 + 3\varrho_3 + 3\varrho_4 + 3\varrho_5 + 3\varrho_6 + 2\varrho_7 + \varrho_8 \\ d_L(y, c_0) &= l - \varrho_0 + \varrho_2 + 2\varrho_3 + 2\varrho_4 + 2\varrho_5 + 2\varrho_6 + \varrho_7. \end{aligned}$$

Alike,

$$d_L(y, c_1) = l - \varrho_1 + \varrho_3 + 2\varrho_4 + 2\varrho_5 + 2\varrho_6 + 2\varrho_7 + \varrho_8,$$

$$d_L(y,c_2) = l - \varrho_2 + \varrho_0 + \varrho_4 + 2\varrho_5 + 2\varrho_6 + 2\varrho_7 + 2\varrho_8,$$
$$d_L(y,c_3) = l - \varrho_3 + 2\varrho_0 + \varrho_1 + \varrho_5 + 2\varrho_6 + 2\varrho_7 + 2\varrho_8,$$
$$d_L(y,c_4) = l - \varrho_4 + 2\varrho_0 + 2\varrho_1 + \varrho_2 + \varrho_6 + 2\varrho_7 + 2\varrho_8,$$
$$d_L(y,c_5) = l - \varrho_5 + 2\varrho_0 + 2\varrho_1 + 2\varrho_2 + \varrho_3 + \varrho_7 + 2\varrho_8,$$
$$d_L(y,c_6) = l - \varrho_6 + 2\varrho_0 + 2\varrho_1 + 2\varrho_2 + 2\varrho_3 + \varrho_4 + \varrho_8,$$
$$d_L(y,c_7) = l - \varrho_7 + \varrho_0 + 2\varrho_1 + 2\varrho_2 + 2\varrho_3 + 2\varrho_4 + \varrho_5,$$
$$d_L(y,c_8) = l - \varrho_8 + \varrho_1 + 2\varrho_2 + 2\varrho_3 + 2\varrho_4 + 2\varrho_5 + \varrho_6.$$

Then, $d_L(y, C_A) = \min\{d_L(x, c_i) | i = 0 \text{ to } 8\} \leq 2l$ and $r_L(C_A) \leq 2l$.

If $y_1 \in \mathbb{Z}^l$, whereas $y_1 = \overbrace{00\cdots0}^{k}\overbrace{11\cdots1}^{k}\overbrace{22\cdots2}^{k}\overbrace{33\cdots3}^{k}\overbrace{44\cdots4}^{k}\overbrace{55\cdots5}^{k}\overbrace{66\cdots6}^{k}\overbrace{77\cdots7}^{k}\overbrace{88\cdots8}^{l-8k}$, here $k = \lfloor\frac{l}{3^2}\rfloor$. Thus, $d_L(y_1, c_i) = 12k,\ i = 0$ to 8 and $r_L(C_A) \geq \min\{d_L(y_1, c_i)|\ i = 0 \text{ to } 8\} \geq 2l$ and hence, $r_L(C_A) = 2l$.

Let $y = \overbrace{33\cdots3}^{\frac{l}{2}}\overbrace{000\cdots0}^{\frac{l}{2}} \in \mathbb{Z}^l$. The code $C_B = \{\gamma(33\cdots3) \mid \gamma \in \mathbb{Z}^l\}$ and it is generated by Type-B. Thus, $r_L(C_B) \geq 2l$.

If $y \in \mathbb{Z}^l$ be any codeword and take y has ϱ_i links i's, with $\sum_i \varrho_i = l$, where $i = 0$ to 8. Then, $r_L(C_B) \leq 2l$.

$\square$

Theorem 3.2. *For* $R_d(C) = \max_{w\in\mathbb{Z}^n}\{\min_{c\in C}\{d(w,c)\}\}$, *where* $d =$ *{ Chinese Euclidean weight, Gray weight and Bachoc weight }.*

1. $R_{CE}(C_A) = \frac{20l}{9}, \frac{3n}{2} \leq R_{CE}(C_B) \leq 2l,$
2. $R_G(C_A) = \frac{4l}{3}, R_G(C_B) = l$ *and*
3. $R_B(C_A) = \frac{4l}{3}, \frac{3l}{2} \leq R_B(C_{B^*}) \leq 2l,$ *where* $B^* =$ *Type-B and* l *is a length of code in Type A and Type B.*

Proof. The methods of proof is follows from Theorem 3.1, by using the Type A and Type B with different weight, such as $w_{CE}(x), w_G(x)$, and $w_B(x)$. $\square$

4. Same size of length in Block repetition code

Let BRC^{2l} be a Block Repetition Code with length $2l$ and its generated by $G_{AB} = [\overbrace{11\cdots1}^{l}\overbrace{33\cdots3}^{l}]$ is size of length(l) for each block and the parameters of BRC^{2l} code is an $[2l, 1, 3l, 3l, 3l, 3l]$.

Theorem 4.1.

1. $R_L(BRC^{2l}) = 4l$,
2. $R_{CE}(BRC^{2l}) = \frac{38l}{9}$,
3. $R_G(BRC^{2l}) = \frac{7l}{3}$ *and*
4. $R_B(BRC^{2l}) = \frac{8l}{3}$.

Proof. Generator matrix G_{AB} and [13] and by using theorem 3.1, then

$$R_L(BRC^{2l}) \geq 4l. \tag{4.1}$$

Consider $y = (y_1 \mid y_2) \in \mathbb{Z}^{2l}$, where $y_1, y_2 \in \mathbb{Z}^{2l}$ and also take in y_1, ϱ_j appears j's, and in y_2, ϱ_j appears j's, with $\sum\limits_j r_j = \sum\limits_j s_j = l$ and $c_j \in \{\gamma(G_{AB}) | \gamma \in \mathbb{Z}^{2l}\}, j = 0$ to 8.

Then, $d_L(y, BRC^{2l}) = \min\{d_L(y, c_j)|j = 0 \text{ to } 8\}$ is less than or equal to $2l + 2l = 4l$. Thus, $d_L(y, BRC^{2l}) \leq 4l$. Hence,

$$R_L(BRC^{2l}) \leq 4l \tag{4.2}$$

By (4.1) and (4.2), thus $R_L(BRC^{2l}) = 4l$.

The remaining Proof of the Theorem 4.1 is pursue from first part. □

Corollary 4.2. *Let*

$$G_A = [\overbrace{11\cdots1}^{l}\overbrace{22\cdots2}^{l}\overbrace{44\cdots4}^{l}\overbrace{55\cdots5}^{l}\overbrace{77\cdots7}^{l}\overbrace{88\cdots8}^{l}] \tag{4.3}$$

is a Type A with unit element in $\mathbb{Z}$. *Then,*

- $R_L(BRC^{6l}) = 12l$,
- $R_{CE}(BRC^{6l}) = \frac{40l}{3}$,
- $R_G(BRC^{6l}) = 8l$ *and*
- $R_B(BRC^{6l}) = 8l$.

Proof. From (4.3) and use to Theorem 3.1, 3.2 and 4.1. □

Corollary 4.3. *Let*

$$G_B = [\overbrace{33\cdots3}^{l}\overbrace{66\cdots6}^{l}] \tag{4.4}$$

is a Type B with zero divisor element in $\mathbb{Z}$. *Then,*

- $R_L(BRC^{2l}) = 4l$,
- $3l \leq R_{CE}(BRC^{2l}) \leq 4l$,
- $R_G(BRC^{2l}) = 2l$ *and*
- $3l \leq R_B(BRC^{2l}) \leq 4l$.

Proof. In (4.4) is apply to Theorem 3.1, 3.2 and 4.1. □

5. Different size of the length for Block repetition code

Let

$$G_{AB} = [\overbrace{11\cdots 1}^{k_1}\overbrace{33\cdots 3}^{k_2}] \tag{5.1}$$

be the generated matrix for the two various block repetition code for a size of length is k_1, k_2 and it is denoted by $BRC^{k_1+k_2}$. The parameters of $BRCp^{k_1+k_2}$ code is an $[k_1+k_2, 1, \min\{3k_1, k_1+3k_2\}, \min\{k_1, k_1+k_2\}, \min\{3k_1, k_1+3k_2\}, \min\{3k_1, k_1+3k_2\}, \min\{3k_1, 2k_1+3_2\}]$.

Theorem 5.1.

- $R_L(BRC^k) = 2k,$
- $R_{CE}(BRC^k) = \frac{20k_1}{9} + 2k_2,$
- $R_G(BRC^k) = \frac{4k}{3}$ *and*
- $R_B(BRC^k) = \frac{4k}{3}$, *there with* $k = \sum\limits_{i=1}^{2} k_i.$

Proof. A generator matrix (5.1), use to Theorem 4.1 and apply the two different size of length(k_1, k_2). □

Corollary 5.2. *Let*

$$G_B = [\overbrace{33\cdots 3}^{k_1}\overbrace{66\cdots 6}^{k_2}] \tag{5.2}$$

is a Type B with zero divisor element and two distinct length(k_1, k_2) *in* $\mathbb{Z}$. *Then*

- $R_L(BRC^k) = 2k,$
- $\frac{3k}{2} \le R_{CE}(BRC^k) \le 2k,$
- $R_G(BRC^k) = k$ *and*
- $\frac{4k}{3} \le R_B(BRC^k) \le 2k$, *here* $k = \sum\limits_{i=1}^{2} k_i.$

Proof. In (5.2) by two distinct length(k_1, k_2) and different weights in put to Theorem 5.1. □

Corollary 5.3. *Let*

$$G_A = [\overbrace{11\cdots 1}^{k_1}\overbrace{22\cdots 2}^{k_2}\overbrace{44\cdots 4}^{k_3}\overbrace{55\cdots 5}^{k_4}\overbrace{77\cdots 7}^{k_5}\overbrace{88\cdots 8}^{k_6}]. \tag{5.3}$$

be a Type A with unit element and alternate size of length in $\mathbb{Z}$. *Then*

- $R_L(BRC^k) = 2k$,
- $R_{CE}(BRC^k) = \frac{20k}{9}$,
- $R_G(BRC^k) = \frac{4k}{3}$ *and*
- $R_B(BRC^k) = \frac{4k}{3}$, *where* $k = \sum_{i=1}^{6} k_i$.

Proof. In (5.3) with alternate size of length and also weight is apply to Theorem 5.1. $\square$

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Chella Pandian P.
email: `chellapandianpc@gmail.com`, `chella@andavancollege.ac.in`
ORCID: 0000-0002-4112-868X
Department of Mathematics
Srimad Andavan Arts and Science College (Auto.)
Tiruchirappalli - 620 005
Tamilnadu
INDIA

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