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On the Analytic α-Lipschitz Vector-Valued Operators

Abbasali Shokri

Abstract: Let (X, d) be a non-empty compact metric space in $\mathbb{C}$, $(B, \parallel . \parallel)$ be a commutative unital Banach algebra over the scalar field $\mathbb{F}(= \mathbb{R}$ *or* $\mathbb{C})$ and $\alpha \in \mathbb{R}$ with $0 < \alpha \leq 1$. In this work, first we define the analytic α-Lipschitz B-valued operators on X and denote the Banach algebra of all these operators by $Lip_A^\alpha(X, B)$. When $B = \mathbb{F}$, we write $Lip_A^\alpha(X)$ instead of $Lip_A^\alpha(X, B)$. Then we study some interesting results about $Lip_A^\alpha(X, B)$, including the relationship between $Lip_A^\alpha(X, B)$ with $Lip_A^\alpha(X)$ and B, and also characterize the characters on $Lip_A^\alpha(X, B)$.

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1. Introduction

Throughout this paper, let (X, d) be a compact metric space in $\mathbb{C}$, $(B, \parallel . \parallel)$ be a commutative unital Banach algebra over the scalar field $\mathbb{F}(= \mathbb{R}$ *or* $\mathbb{C})$ with unit $\mathbf{e}$, $C(X, B)$ be the set of all B-valued continuous operators on X, and also $\alpha \in \mathbb{R}$ with $0 < \alpha \leq 1$.

The *dual space* of B is the vector space B^* whose elements are the continuous linear functionals on B. The set of all multiplicative functionals on B is called *spectrum* of B; we denote it by $\sigma(B)$. Suppose that throughout this article, $\Lambda \in \sigma(B)$ is arbitrary and fixed. Since $\sigma(B)$ is a subset of the closed unit ball of B^*, $\parallel \Lambda \parallel$ is bounded, where

$$\parallel \Lambda \parallel = \sup\{ \mid \Lambda x \mid \; : \; x \in B \; , \; \parallel x \parallel \leq 1 \}.$$

When $B = \mathbb{F}$, take Λ as the identity function $\Lambda x = x$.

Consider the set Y as follows

$$Y := \{(x, y) \ : \ x, y \in X \ , \ x \neq y\}.$$

For an operator $f : X \to B$ and any $(x, y) \in Y$ define

$$L_f^{\alpha}(x, y) := \frac{\Big|\big(\Lambda o f\big)(x) - \big(\Lambda o f\big)(y)\Big|}{d^{\alpha}(x, y)} \ ,$$

where $d^{\alpha}(x, y) = \big(d(x, y)\big)^{\alpha}$ and $0 < \alpha \leq 1$. Now define

$$p_{\alpha}(f) := \sup_{x \neq y} \ L_f^{\alpha}(x, y) \ , \ 0 < \alpha \leq 1,$$

which is called the *Lipschitz constant* of f. Also for $0 < \alpha \leq 1$ define

$$Lip^{\alpha}(X, B) := \{f \in C(X, B) \ : \ p_{\alpha}(f) < +\infty\},$$

and for $0 < \alpha < 1$ define

$$lip^{\alpha}(X, B) := \{f \in Lip_{\alpha}(X, B) \ : \ \lim_{d(x,y) \to 0} L_f^{\alpha}(x, y) = 0\}.$$

The elements of $Lip^{\alpha}(X, B)$ and $lip^{\alpha}(X, B)$ are called *big* and *little* α-Lipschitz B-valued operators, respectively.

Now, for each $\lambda \in \mathbb{F}$, $x \in X$ and $f, g \in C(X, B)$ define

$$\big(f + g\big)(x) := f(x) + g(x) \ , \ \big(\lambda f\big)(x) := \lambda f(x) \ ,$$

and the uniform norm $\| \ . \ \|_{\infty}$ on $C(X, B)$ by

$$\| \ f \ \|_{\infty} := \sup_{x \in X} \| \ f(x) \ \| \quad ; \quad f \in C(X, B).$$

Also for any $f \in Lip^{\alpha}(X, B)$ define

$$\| \ f \ \|_{\alpha} := p_{\alpha}(f) + \| \ f \ \|_{\infty} \ .$$

It is easy to see that $\big(C(X, B), \| \ . \ \|_{\infty} \ \big)$ becomes a Banach algebra over $\mathbb{F}$.

Cao, Zhang and Xu in [6] proved that $\big(Lip^{\alpha}(X, B), \| \ . \ \|_{\alpha} \ \big)$ is a Banach space over $\mathbb{F}$ and $\big(lip^{\alpha}(X, B), \| \ . \ \|_{\alpha} \ \big)$ is a closed linear subspace of $\big(Lip^{\alpha}(X, B), \| \ . \ \|_{\alpha} \ \big)$ when B is a Banach space. We also studied some of the properties of these algebras in [14-17] when B is a commutative unital Banach algebra.

Note that for $\alpha = 1$ and $B = \mathbb{F}$, the space $Lip^{1}(X, \mathbb{F})$ consisting of all Lipschitz functions from X into $\mathbb{F}(= \mathbb{R} \ or \ \mathbb{C})$ has a series of interesting and important properties, which has been studied by many mathematicians, including the first of them Sherbert [13]. In [7, 18] some properties of Lipschitz scalar-valued functions are mentioned.

Let D be an open subset of X. An operator f of D into B is said to be *analytic* on D if, for every continuous linear functional $\phi \in B^*$, the scalar-valued function $\phi o f$

is analytic on D in the usual sense. Note that we do not require D to be connected. For a full discussion of analytic complex-valued and vector-valued functions, see [2, 7]. The algebra of all continuous B-valued operators on X whose analytic in interior X is denoted by $A(X, B)$. We write $A(X)$ instead of $A(X, \mathbb{F})$ ($\mathbb{F} = \mathbb{R}$ *or* $\mathbb{C}$). Some of the properties of these algebras have been studied by certain mathematicians, see [1, 3-5, 8-11].

Finally, in this article, we introduce the analytic α-Lipschitz B-valued operator algebras $Lip_A^\alpha(X, B)$ and we characterize their characters, also we study the relationship between of $Lip_A^\alpha(X, B)$ and B. We prove the main results of the article in several theorems.

2. Lip-analytic Operators

In this section, we introduce the analytic α-Lipschitz vector-valued operator algebras $Lip_A^\alpha(X, B)$, and we study some of their properties.

We write $C(X)$ and $Lip^\alpha(X)$ instead of $C(X, \mathbb{F})$ and $Lip^\alpha(X, \mathbb{F})$ respectively. By the Stone-Weierstrass theorem, we have

Theorem 2.1. [7]. *$A(X)$ is uniformly dense in $C(X)$.*

It is obvious that $A(X, B)$ is a subalgebra of $C(X, B)$. We have

Theorem 2.2. *$A(X, B)$ is uniformly dense in $C(X, B)$.*

Proof. Let $\epsilon > 0$ and $f \in C(X, B)$ be arbitrary. We show that there exists $g \in A(X, B)$ such that $\| f - g \|_\infty < \epsilon$. Since $f \in C(X, B)$, $\Lambda o f \in C(X)$. Then by Theorem 2.1, there is $h \in A(X)$ such that $\| \Lambda o f - h \|_\infty < \epsilon$. So

$$\sup_{x \in X} \Big| (\Lambda o f)(x) - h(x) \Big| < \epsilon.$$

Since $\Lambda(\mathbf{e}) = 1$, $h(x) = \Lambda(h(x)\mathbf{e})$ for all $x \in X$. Then

$$\sup_{x \in X} \Big| \Lambda(f(x)) - \Lambda(h(x)\mathbf{e}) \Big| < \epsilon.$$

Hence

$$\sup_{x \in X} \Big| \Lambda\big((f - h.\mathbf{e})(x)\big) \Big| < \epsilon.$$

Since $\Lambda \in \sigma(B)$ is arbitrary, $\sup_{x \in X} \| (f - h.\mathbf{e})(x) \| < \epsilon$. Thus $\| f - h.\mathbf{e} \|_\infty < \epsilon$. Now, take $g := h.\mathbf{e}$. Since $h \in A(X)$ and $\mathbf{e} \in B$, $g \in A(X, B)$. Therefore we conclude that $\| f - g \| < \epsilon$ where $g \in A(X, B)$. $\square$

We have the similar Theorem 2.1 for the algebra of Lipschitz scalar-valued functions:

Theorem 2.3. [18]. *$Lip^\alpha(X)$ is uniformly dense in $C(X)$.*

Theorem 2.4. *$Lip^\alpha(X,B)$ is uniformly dense in $C(X,B)$.*

Proof. Let $\epsilon > 0$ and $f \in C(X,B)$ be arbitrary. We show that there exists $h \in Lip^\alpha(X,B)$ such that $\| h - f \|_\infty < \epsilon$. Since $f \in C(X,B)$, $\Lambda o f \in C(X)$. So by Theorem 2.3, there exists $g \in Lip^\alpha(X)$ such that $\| g - \Lambda o f \|_\infty < \epsilon$. Define

$$\eta : \mathbb{C} \to B$$

$$\eta(\lambda) := \lambda \mathbf{e}.$$

Since g is continuous, $\eta o g$ is continuous. Also

$$\begin{aligned} p_\alpha(\eta o g) &= \sup_{x \neq y} L^\alpha_{\eta o g}(x,y) \\ &= \sup_{x \neq y} \frac{\| (\eta o g)(x) - (\eta o g)(y) \|}{d^\alpha(x,y)} \\ &= \sup_{x \neq y} \frac{\| g(x)\mathbf{e} - g(y)\mathbf{e} \|}{d^\alpha(x,y)} \quad (\| \mathbf{e} \| = 1) \\ &\leq p_\alpha(g) < \infty. \end{aligned}$$

So $\eta o g \in Lip^\alpha(X,B)$. Set $h := \eta o g$. Now we show that $\| h - f \|_\infty < \epsilon$. Since $\Lambda(\mathbf{e}) = 1$, for all $x \in X$ we have

$$\begin{aligned} \big|\Lambda\big(g(x)\mathbf{e} - f(x)\big)\big| &= \big|g(x) - (\Lambda o f)(x)\big| \\ &\leq \| g - \Lambda o f \|_\infty \\ &< \epsilon. \end{aligned}$$

This implies that

$$\big|\Lambda\big(\eta o g - f\big)(x)\big| < \epsilon \ , \ x \in X.$$

Since $\Lambda \in \sigma(B)$ is arbitrary, $\| (\eta o g - f)(x) \| < \epsilon$ for all $x \in X$. Consequently, $\| \eta o g - f \|_\infty < \epsilon$ or $\| h - f \|_\infty < \epsilon$. This completes the proof. □

Corollary 2.5. *By using Theorems 2.2 and 2.4, each element of $A(X,B)$ can be approximated by elements of $Lip^\alpha(X,B)$ with sup-norm. Also each element of $Lip^\alpha(X,B)$ can be approximated by elements of $A(X,B)$ with sup-norm.*

Definition 2.6. Let D be an open subset of X. An operator f of D into B is said to be Lip-analytic on D if $f \in Lip^\alpha(X,B) \cap A(X,B)$.

The algebra of all Lip-analytic B-valued operators on X whose analytic in interior X is denoted by $Lip^\alpha_A(X,B)$. When $B = \mathbb{F}$, we write $Lip^\alpha_A(X)$ instead of $Lip^\alpha_A(X,B)$.

By Theorems 2.2 and 2.4, we can prove that:

Theorem 2.7. *$Lip^\alpha_A(X,B)$ is uniformly dense in $C(X,B)$.*

Let E_1 and E_2 be linear spaces. From [12], a *tensor product* of E_1 and E_2 is a pair (T,τ), where T is a linear space and $\tau : E_1 \times E_2 \to T$ is a bilinear map with the following (universal) property: For each linear space F and for each bilinear map $V : E_1 \times E_2 \to F$, there is a unique linear map $U : T \to F$ such that $V = Uo\tau$.
We shall also use the standard notation for tensor products, we write $E_1 \otimes E_2$ for T and $x_1 \otimes x_2 = \tau(x_1, x_2)$ for $x_1 \in E_1$ and $x_2 \in E_2$. If $Z \in E_1 \otimes E_2$, then there is $m \in \mathbb{N}$, and for each $j = 1, 2$ there are $x_j^{(1)}, ..., x_j^{(m)} \in E_j$ such that $Z = \sum_{k=1}^{m} x_1^{(k)} \otimes x_2^{(k)}$.

Let E_1 and E_2 be Banach spaces with dual spaces E_1^* and E_2^* . Then we define for $Z \in E_1 \otimes E_2$

$$\| Z \|_\epsilon = \sup \Big\{ \big| \langle Z, \phi_1 \otimes \phi_2 \rangle \big| \ : \ \phi_j \in N_1[0, E_j^*] \ , \ j = 1, 2 \Big\},$$

where

$$Z = \sum_{k=1}^{m} x_1^{(k)} \otimes x_2^{(k)} \ ; \ \big(m \in \mathbb{N}, \ x_j^{(k)} \in E_j, \ j = 1, 2, \ 1 \leq k \leq m \big),$$

and

$$\begin{aligned} \langle \ Z, \phi_1 \otimes \phi_2 \ \rangle &= \langle \ \sum_{k=1}^{m} x_1^{(k)} \otimes x_2^{(k)}, \phi_1 \otimes \phi_2 \ \rangle \\ &= (\phi_1 \otimes \phi_2) \Big(\sum_{k=1}^{m} x_1^{(k)} \otimes x_2^{(k)} \Big) \\ &= \sum_{k=1}^{m} (\phi_1 \otimes \phi_2) \big(x_1^{(k)} \otimes x_2^{(k)} \big) \\ &= \sum_{k=1}^{m} \phi_1 \big(x_1^{(k)} \big) \ \phi_2 \big(x_2^{(k)} \big), \end{aligned}$$

and $N_1[0, E_j^*]$ is closed ball in E_j^* with radius 1 centered at 0 . We call $\| \ . \ \|_\epsilon$ the *injective norm* on $E_1 \otimes E_2$.

Let $(E_1, \| \ . \ \|_1)$ and $(E_2, \| \ . \ \|_2)$ be Banach spaces. Then their *injective tensor product* $E_1 \check{\otimes} E_2$ is the completion of $E_1 \otimes E_2$ with respect to $\| \ . \ \|_\epsilon$. For every $Z \in E_1 \check{\otimes} E_2$, we have

$$\| Z \|_\epsilon = \sup \Big\{ \ \| \big(id \otimes \phi \big)(Z) \|_1 \ \ : \ \phi \in N_1[0, E_2^*] \Big\},$$

where

$$\big(id \otimes \phi \big)(a \otimes b) = a\phi(b) \ ; \ (a \in E_1 \ , \ b \in E_2).$$

Definition 2.8. Let E_1 and E_2 be Banach spaces. A norm $\| \ . \ \|$ on $E_1 \otimes E_2$ is called a cross norm if

$$\| x_1 \otimes x_2 \| = \| x_1 \| \| x_2 \| \quad (x_1 \in E_1 \ , \ x_2 \in E_2).$$

Proposition 2.9. [12]. *Let E_1 and E_2 be Banach spaces. Then $\| \ . \ \|_\epsilon$ is a cross norm on $E_1 \otimes E_2$.*

3. The Main Results

In this section, we present the main results of the article.

Theorem 3.1. *$Lip_A^\alpha(X,B)$ is isometrically isomorphic to $Lip_A^\alpha(X)\check{\otimes}B$.*

Proof. It is straightforward to prove that the mapping

$$Lip_A^\alpha(X)\times B\to Lip_A^\alpha(X,B),\quad (f,b)\longmapsto fb \tag{3.1}$$

is bilinear. So from the defining property of the algebraic tensor product $Lip_A^\alpha(X)\otimes B$, it follows that (1) extends to a linear map

$$S: Lip_A^\alpha(X)\otimes B\longrightarrow Lip_A^\alpha(X,B)$$

$$S(f\otimes b):=fb\ ,$$

where

$$(fb)(x):=f(x)b\ ;\ (x\in X).$$

Then

$$\begin{aligned}\parallel S(f\otimes b)\parallel_\alpha &= \parallel fb\parallel_\alpha \;=\; \parallel fb\parallel_\infty + p_\alpha(fb)\\ &= \parallel f\parallel_\infty\parallel b\parallel + p_\alpha(f)\parallel b\parallel\\ &= (\parallel f\parallel_\infty + p_\alpha(f))\parallel b\parallel\\ &= \parallel f\parallel_\alpha\parallel b\parallel\\ &= \parallel f\otimes b\parallel_\epsilon\ .\end{aligned}$$

Therefore S is an isometry and thus injective with closed range. It remains to be shows that it has dense range as well.

Let $f\in Lip_A^\alpha(X,B)$ and $\epsilon>0$. Being the continuous image of a compact space, $K:=f(X)\subset B$ is compact. We may thus find $b_1,...,b_n\in B$ such that $K\subset\cup_{i=1}^n N(b_i,\epsilon)$, where $N(b_i,\epsilon)$ is a neighborhood with radius ϵ centered at b_i. Let $U_j:=f^{-1}\big(N(b_j,\epsilon)\big)$ for $j=1,...,n$. Choose $f_1,...,f_n\in Lip_A^\alpha(X,B)$ such that $supp(f_j)\subset U_j$ for j = 1 , ..., n, and $\Lambda o(\sum_{i=1}^n f_i)=1$. Then for every $x\in X$ we have

$$\begin{aligned}\parallel \big(f-\sum_{i=1}^n S(\Lambda o f_i\otimes b_i)\big)(x)\parallel &= \parallel \big(f-\sum_{i=1}^n(\Lambda o f_i)b_i\big)(x)\parallel\\ &= \parallel f(x)-\sum_{i=1}^n(\Lambda o f_i)(x)b_i\parallel\\ &= \parallel f(x)\big(\Lambda o(\sum_{i=1}^n f_i)\big)(x)-\sum_{i=1}^n(\Lambda o f_i)(x)b_i\parallel\\ &= \parallel f(x)\sum_{i=1}^n(\Lambda o f_i)(x)-\sum_{i=1}^n(\Lambda o f_i)(x)b_i\parallel\end{aligned}$$

$$= \quad \| \sum_{i=1}^{n} (\Lambda o f_i)(x)\big(f(x) - b_i\big) \|$$
$$\leq \quad \sum_{i=1}^{n} | (\Lambda o f_i)(x) | \| f(x) - b_i \| .$$

It easy to see that the right hand side of the above relation is less than ϵ. So we conclude that $\overline{R_S} = Lip_A^\alpha(X, B)$. This completes the proof. $\square$

With an argument similar to the proof of Theorem 3.1, we can prove that:

Theorem 3.2. *$A(X, B)$ is isometrically isomorphic to $A(X)\check{\otimes}B$.*

Define the canonical embedding

$$j : Lip_A^\alpha(X) \to Lip_A^\alpha(X, B)$$
$$j(h) := h \otimes \mathbf{e} \ ,$$

such that

$$(h \otimes \mathbf{e})(x) := h(x)\mathbf{e} \ ; \ x \in X.$$

By Theorem 3.1, the map j is well defined. Let χ be a arbitrary and fixed character on $Lip_A^\alpha(X, B)$. Then there is $z \in X$ such that χoj is the evaluation at z, indeed $\chi oj = \delta_z$ where $\delta_z(f) = f(z)$.

Define $\varphi(\omega) := \omega - z, (\omega \in X)$. It is clear that $\varphi \in A(X)$, and we have

$$p_\alpha(\varphi) = \sup_{x \neq y} \frac{| \varphi(x) - \varphi(y) |}{| x - y |^\alpha} \quad = \quad \sup_{x \neq y} \frac{| (x - z) - (y - z) |}{| x - y |^\alpha}$$
$$= \quad \sup_{x \neq y} | x - y |^{1-\alpha} < \infty.$$

So $\varphi \in Lip^\alpha(X)$, and consequently $\varphi \in Lip_A^\alpha(X)$.

Now consider

$$I := \{f \in Lip_A^\alpha(X, B) \ : \ f(z) = 0\}.$$

It is obvious that I is nonempty and an ideal in $Lip_A^\alpha(X, B)$.

Theorem 3.3. *I is contained in the kernel of χ.*

Proof. Let $f \in I$ be arbitrary. Then $f \in A(X, B)$. So f has a Taylor series expansion $f(\omega) = \Sigma_{n=1}^{\infty} \frac{f^{(n)}(z)}{n!}(\omega - z)^n$ around z. Define

$$g(\omega) := \begin{cases} \frac{f(\omega)}{\omega - z} & ; \quad \omega \neq z \ , \\ f^{'}(z) & ; \quad \omega = z. \end{cases}$$

It is clear that $\Lambda o g$ is analytic in the interior of X, so $g \in A(X, B)$. For $\omega = z$, it is obvious that $g \in Lip_A^{\alpha}(X, B)$, and for $\omega \neq z$ we have

$$f(\omega) = (\omega - z)g(\omega) = \varphi(\omega)g(\omega).$$

It can be easily proved that $g \in Lip_A^{\alpha}(X, B)$. Then for every $\omega \in X$ with $\omega \neq z$, we have

$$\begin{aligned} f(\omega) &= \varphi(\omega)g(\omega) = \varphi(\omega)\mathbf{e}g(\omega) \\ &= (\varphi \otimes \mathbf{e})(\omega)g(\omega) = \big((\varphi \otimes \mathbf{e})g\big)(\omega) \\ &= \big(j(\varphi)g\big)(\omega). \end{aligned}$$

So $f = j(\varphi)g$. Therefore

$$\begin{aligned} \chi(f) &= \chi\big(j(\varphi)g\big) = \chi\big(j(\varphi)\big)\chi(g) \\ &= \big(\chi o j\big)(\varphi)\chi(g) = \delta_z(\varphi)\chi(g) \\ &= \varphi(z)\chi(g) = 0 \times \chi(g) = 0. \end{aligned}$$

So $f \in ker\chi$, and that means $I \subset ker\chi$. This completes the proof. □

Theorem 3.4. *Every character χ on $Lip_A^{\alpha}(X, B)$ is of form $\chi = \psi o \delta_z$ for some character ψ on B and some $z \in X$, where $\delta_z(f) = f(z)$.*

Proof. Let χ be an arbitrary character on $Lip_A^{\alpha}(X, B)$. Then there is $z \in X$ such that $\chi o j$ is the evaluation at z, indeed $\chi o j = \delta_z$ where $\delta_z(f) = f(z)$. Define

$$I := \{f \in Lip_A^{\alpha}(X, B) \ : \ f(z) = 0\}.$$

By Theorem 3.3, I is contained in the kernel of χ. It is clear that $ker\delta_z = I$. Therefore $ker\delta_z \subset ker\chi$. We obtain the desired factorization $\chi = \psi o \delta_z$ for some character ψ on B. □

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Abbasali Shokri
email: `Ab.Shokri@iau.ac.ir, shokrimath@yahoo.com`
ORCID: 0000-0002-1759-1439
Department of Mathematics,
Ahar Branch, Islamic Azad University,

Ahar,
IRAN

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