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ON THE TWO POSSIBLE TYPES OF THZ GENERATORS

This paper is the review of our study published earlier in *Acta Phys. Polon. A* **121**, 522 (2012) [7], *Phys Lett A* **378**, 1364 (2014) [25], and [1]. It's aim is to pay attention to the new possibilities related to producing the new types of THz generators. The suggested effects are the result of combining two effects: Gunn-effect in a material such as GaAs and undulator-like radiation, or 'pumping wave' acting on the electrons which is the result of undulator field, while the second is the backward effect of radiation which is produced by electrons moving within such a micro-undulator. It is very probable that the effects can be used to develop a new semiconductor-based room temperature source of the THz-radiation.

Keywords: undulator, synchrotron radiation, Gunn effect, synchronization, Dicke model, superradiance

1. INTRODUCTION

Nowadays there is a growing interest in the development of the new sources of THz-radiation because of variety of its possible applications ranging from security service to biochemistry and medicine [2]. There are many propositions concerning possible schemes of THz generation/detection, broad-band as well as narrow-band, based on optical rectification, photoconductive effect, parametric conversion (for the review see [3,4]). It seems, that the existing modern fabrication techniques enable to develop the room temperature sources of THz radiation, based on the 'micro-undulator' which would be able to generate in this spectral region. The paper is organized as follows: for the reader's convenience, in the *Introduction* we outline briefly the basic information regarding undulator radiation and Gunn effect which is of use in further discussion. In Section 2 we discuss the concept of the type I THz-generator and the 'N^2–effect' in semiconductor microstructure with greating, while in Section 3 we discuss the type II THz-generator. The discussion is preceded by some information related to the Dicke model.

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1.1. Undulator radiation

Undulator is one among the three well-known sources of synchrotron radiation, alongside cyclotron and wiggler. It is made of periodically aligned with the space period *a*, dipole magnets whose magnetic field changes along the device axis. Electron bunch moving along the axis is forced to move along the snake-like trajectory. Since the movement of charges along the curving trajectory is the movement with acceleration, the electron bunch has to emit the electromagnetic radiation. In the reference frame of the bunch moving with the velocity v that we assume to be relativistic, the undulator period becomes a/γ, where γ is the Lorentz-factor equal to $1/\sqrt{1-v^2/c^2}$ and *c* is the light velocity in a vacuum. In this reference frame the frequency of electron oscillations is equal to $\omega' = 2\pi c/a'$. From Lorentz transformation it follows that in the reference frame of undulator the oscillation frequency is given by

$$\omega = \frac{\omega'}{\gamma\left(1-\frac{v}{c}\cos\vartheta\right)} = \frac{2\pi}{a}\frac{1}{\left(1-\frac{v}{c}\cos\vartheta\right)}, \tag{1}$$

where ϑ is the observation angle. A more detailed analysis that takes into account the form of the electron trajectory and other factors lead to the following formula for the wavelength of the emitted wave

$$\lambda = \frac{a}{2\gamma^2}\left(1+\frac{K^2}{2}+\gamma^2\vartheta^2\right), \tag{2}$$

where the dimensionless parameter *K* called *undulator parameter* is equal

$$K = \frac{eB_0 a}{2\pi m_e c}, \tag{3}$$

where B_0 is the maximum value of the magnetic field on the undulator axis, *e* and m_e are as usual, the electron charge and electron mass, respectively. It is worth noticing that formula (3) describes only the first harmonic, the higher harmonics, for instance *m*-th harmonic is given by

$$\lambda_m = \frac{\lambda}{m}. \tag{4}$$

The width of the harmonic of the emitted radiation is equal to

$$\frac{\Delta\lambda}{\lambda} \sim \frac{1}{Mm}, \tag{5}$$

where M is the number of magnets in the undulator. Note that radiation wavelength or frequency can be adjusted in some range by means of changing of total electron energy $\gamma m_e c^2$ and/or by changing parameter K and namely the value of K distinguish undulator ($K \leq 1$) from wiggler ($K >> 1$). The total power of radiation emmited during one period of oscillations per one electron can be estimated as [5]

$$P = \frac{e^4 \gamma^2 B_0^2}{12\pi\varepsilon_0 c m_e^2}, \tag{6}$$

where ε_0 is the dielectric constant of vacuum.

1.2. Gunn effect

The Gunn effect is a high-frequency oscillations of electrical current flowing through certain semiconductors, such as GaAs, InSb and a few others. The effect was discovered by J.B. Gunn in the early 1960s. In such materials when electric field is applied to the sample, one can observe the decrease of electron mobility as the electron field goes above some threshold voltage, characteristic for the chosen material. Macroscopically it manifests itself as the decreasing of current with the increasing of applied voltage that is, in appearing of negative differential resistance. The diode is made of such materials and put into the electron circuit produces the current oscillations at microwave frequencies and above by applying DC voltage and biasing the device into negative resistance region. In order to achieve such an effect, the material the diode is made of, has to have specific band structure. Semiconductors made of elements belonging to III and V groups are often the materials of *direct* energy gap, i.e. their conduction band global minimum is in the center of Brillouin zone (so called Γ point), i.e. there are also *satellite valleys* located at the points L and X in [111] and [100] directions, respectively. In such materials at small values of applied electric field the majority of electrons in conduction band is in the central valley at Γ point with their quasi-momentum in the vicinity of $k = 0$. As the electric field increases, electron energy E also increases and electrons are able to occupy the states of higher energy. When their energy becomes at least equal to the energy difference between local minimum at L-valley and global minimum at Γ-valley, electrons can occupy the states in the vicinity of L-valley. Since the electron effective mass m^* depends on the second derivative d^2E/dk^2 and defines the curvature of energy surface $E(k)$ as

$$m^* = \frac{\hbar^2}{\frac{d^2E(k)}{dk^2}},$$

hence in the vicinity of satellite valley L, where the curvature is greater, the electron effective mass in case of GaAs is much greater in L-valley than in the central Γ-valley. As a result, the electron mobility that is inversely proportional to the effective mass, becomes dozens of times smaller in satellite valley than mobility in the central valley and hence, the electron mean velocity also becomes smaller. Macroscopically it manifests itself as decreasing the current density as the electric field becomes greater and the region of negative differential conduction (or negative differential resistivity as well) in the *I*-*V* characteristics (the plot *current vs. voltage*) appears. The appearance of negative differential resistivity leads to the spontaneous formation of *Gunn domains*, i.e. domains of the increased electron density. Assume that the voltage V_0 is applied to the sample of, say GaAs of the length L, so the mean electric field within the sample $E_0=V_0/L$ is greater than the threshold elecrtic field which is needed for the negative differential resistivity to appear. Electrons will move from cathode to anode with the velocity v_3 (see Fig.1). If in some moment of time there was a small perturbation of charge density caused by, for instance, the local variation of temperature or other reasons within the sample at some point, say *A*, the electrons at this point would be under the influence of the electric field E_{L1} and would move with the velocity v_4. In their turn on the electrons at point *B* acts the electric field E_{H1}, and they will move to the anode with the velocity v_2 smaller than v_4. This will lead to the accumulation of electrons between points *A* and *B* and hence, to the greater local charge density.

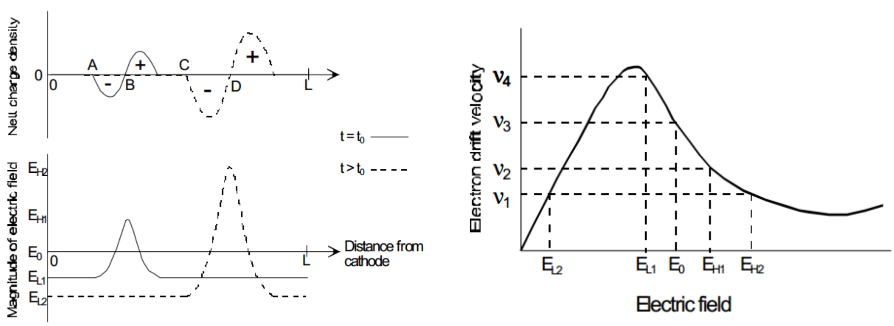

Fig 1. Gunn domains formation

Then, in the area to the right of point *B* there will be less electrons, because they move to the anode with the velosities which are greater than electron velosities at point *B*. It means that local perturbation of charge density leads to the spontaneous formation of the domain with the increased electron density called Gunn domain. It will become larger as it will move towards the anode up to the moment when it will become stable, i.e. when the velocities at point *C* and *D* will be equal. When the domain is already formed, electric field within the sample drops to the value smaller than the threshold value. The last one stops the formation of another domain up to the moment when the first one reaches the anode, then the process can repeat itself.

2. THE CONCEPT OF TYPE I THZ-GENERATOR

2.1. Preliminaries

Suppose we have a micro-structure shown in Fig. 2, which could be grown on a semi-insulating GaAs substrate, with multiple electrodes on both sides of it, top and bottom (see Fig. 2), which create gratings. If the electric bias is applied to the electrodes, a weak periodic potential modulation arises within the GaAs sample and this periodic electric field becomes very similar to the periodic electric field used in some types of undulators.

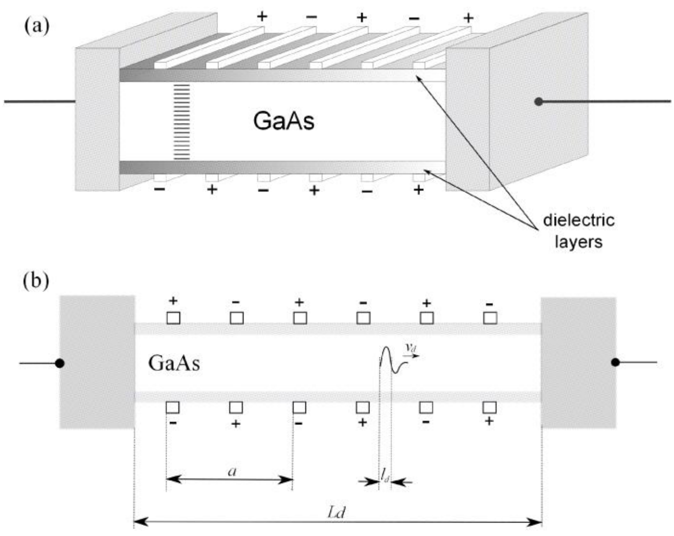

Fig. 2. Undulator microstructure, L_d – the sample length, a – the grating period, v_d – the velocity of the electron bunch

The radiation field produced by the charged particles moving in such structure should consist of narrow spectral lines whose frequencies are described by the formula similar to (1):

$$\omega = \frac{2\pi m v}{2}\left(1 - \frac{v}{c}\cos\vartheta\right)^{-1}, \tag{7}$$

where c is the light velocity in a vacuum, m is an integer and ϑ is the observation (excursion) angle as previously in the expression (2). This is exactly the radiation spectrum emitted by the particles in an undulator. The spectrum described by the formula (7) does not depend on whether the oscillations of moving particles are caused by the periodic magnetic or periodic electric field, since the situation in both cases is similar and, in a sense, both configurations are equivalent. Obviously, assume the electron velocity oscillations in the direction perpendicular to the axis of the structure are caused by the periodically aligned magnets. Then, in the reference frame moving along the structure axis where electron is at rest on average, the undulator field has not only magnetic, but the transverse electric component as well. Thus, it resembles an electromagnetic "pumping wave," while in case of the periodic arrangement of electric field, the undulator field has the transversal magnetic component also resembling electromagnetic "pumping wave." Hence, all the formulae of Section I can be used in case of periodic alignment of electric field substituting $B_0 \rightarrow E_0/c$, where now E_0 is the maximum value of the electric field on the undulator axis. It is worth mentioning that indeed, in most of the cases in undulators the periodic arrangement of magnets is used, however the periodic electric field is also employed, especially in free electron lasers (FEL). The observed spectrum of proposed structure will be composed not of the ideal narrow spectral lines, but of the lines of finite width described by the formula similar to (5). The additional source of spectral lines broadening will be the fluctuations of electron velocities caused by temperature fluctuations, which can be estimated as

$$\frac{\Delta_T \omega}{\omega} = \sqrt{\frac{8k_B T \ln 2}{m_e c^2}}, \tag{8}$$

where T stands for temperature and k_B is Boltzmann constant. An interesting question to be posed is whether one should take into account the higher harmonics of radiation, or is it sufficient to consider only the lowest ones? It turns out that if $v \ll c$ (note that the electron velocity is not greater than 10^7 cm· s^{-1}), the highest harmonics can be neglected. Another argument in favour of considering only the lowest harmonics is that if a function of a single variable $f(t)$ is differentiable everywhere within some closed interval and its first derivative is a function of bounded variance, the coefficients of the Fourier series for $f(t)$ decrease as their index m increases, at least as m^{-2} (see [6]). If instead, the smoothness of the function $f(t)$ is of a higher order, then the Fourier coefficients can decrease even exponentially.

2.2. Cooperative N^2-effect in an ensemble of classical oscillators

The effect results in the radiation intensity proportional to N^2, where N is the number of emitters, and is due to specific phase correlations arising in the ensemble of radiating atoms. Perhaps the first person who mentioned such a possibility was Schiff [8]. He noticed that if N particles moving around the circle have the same speed and instantaneous angular position φ_s, then the formula for radiation intensity contains the factor $\left|\sum_{s=1}^{N}\exp(im\varphi_s)\right|^2$, where m is an integer. If the particles are distributed at random around the orbit, this factor is equal to N and radiation intensity per particle is independent on the other particles. If, however particles are spaced equally around the circle, the expression above equals zero unless m/N is an integer, in which case it is equal to N^2.

Particles' movement in a periodic potential is formally similar to the circular motion and hence, one can state that if the ensemble of N identical oscillators (the source of radiation) emits waves, the amplitudes of generated fields being seen at some distance from the source, can add up to each other in a different way, depending on their relative phases. In general, for more than one emitter, each emitter radiates with its own phase, and these phases are completely random with respect to one another. The total emitted field is $\vec{E}_t = \vec{E}_t^{+} + \vec{E}_t^{-}$, where $\vec{E}_t^{+} = \sum_{r=1}^{N}\vec{E}_r^{+}\exp(i\varphi_r)$, and the sum runs over all emitters in the system. The total radiated intensity is:

$$I_t = \left|\sum_{r=1}^{N}\vec{E}_r^{+}\exp(i\varphi_r)\right|^2 = \sum_{r=1}^{N}\left|\vec{E}_r^{+}\right|^2 + \sum_{r\neq s}\vec{E}_r^{-}\vec{E}_s^{+}\exp(-i(\varphi_r-\varphi_s)).$$

Assuming that the amplitudes of the fields emitted by each oscillator are the same $\left|\vec{E}_r\right|^2 = I$, we get

$$I_t = NI + I\sum_{r\neq s}\exp(-i(\varphi_r-\varphi_s)).$$

For random phases, the second term in the expression above averages to zero, thus the total intensity is just the sum of individual intensities: $I_t = NI$. If however, we could somehow force oscillators to emit with roughly the same phase $\varphi_r \approx \varphi_s$ for all *r* and *s*, than $I_t = NI + N(N-1)I \sim N^2 I$, since usually $N >> 1$. From now on we shall term the proportionality of radiated intensity to N^2 as N^2-effect for short.

The possibility to get the generated power $\sim N^2$ was discussed many times in scientific literature and in different contexts. For example, a similar problem appears in the discussion of the operation modes of FEL (see for instance, [9]). Yet, if FEL is to generate visible light, to create a bunch whose linear size is of a wavelength then it is, mildly speaking, not an easy task. It seems however, that it is quite possible to produce a bunch whose linear size is much smaller than the wavelength corresponding to THz region. This possibility will be discussed in the next section of this paper.

2.3. Gunn effect and undulator-like radiation

Suppose we have a GaAs sample normally used for the fabrication of the Gunn-effect diodes and assume that this Gunn-effect diode structure is equipped with the gratings similar to those presented in Fig. 2. Note that such structure is like the one reported in [10]. Then, due to the gratings on the top of the structure there will be a weak periodic potential modulation within the semiconductor. Suppose now that all other conditions necessary for the Gunn effect to appear are fulfilled. Then, the strong electric field domain moving within the sample is accompanied by the electron 'bunch', whose electron concentration is greater than some threshold concentration and which can be estimated at about 10^{16}cm^{-3}. The thickness of the 'bunch' l_d ranges from 1/10 to 1/30 of the sample length L_d. If we suppose the length of the Gunn-effect diode L_d to be equal to 10^{-3} cm, the thickness of a bunch can be estimated to be $\sim 3 \cdot 10^{-5}$ cm. If we assume the period of grating *a* on a top and bottom of the structure to be about 2×10^{-4} cm, the frequency of the undulator-like radiation produced by the structure, will be about $\omega_0 \approx \frac{2\pi v_d}{a} / (1 - v_d / c) \approx$ 3.14· 10^{11} Hz. (Here we take into account that the speed of the strong field domain v_d is about 10^7 cm·s^{-1} and hence, $v_d / c \approx 10^{-3}$ and radiation frequency practically does not depend on the excursion angle ϑ. The corresponding wavelength is about $\lambda \approx 0.1816$ cm and obviously, $l_d << \lambda$. If the velocity of electrons in a bunch is ultra-relativistic as in the case of FEL or in general, in undulators, the last condition is sufficient for the radiation power to be $\sim N^2$. But since in our case the velocity of electrons is much smaller than *c*, to take into account that the size of the bunch has to be much smaller than the period of grating $l_d << a$ is even more important than to take into account the condition $l_d << \lambda$, as it will be shown in some of the next sections (see also [7]). The last one guarantees that electrons in the bunch will generate the EM waves of the same phase roughly. We have now all reasons to believe that the electron "bunch", i.e. the domain of high electron concentration accompanying the strong field domain in the Gunn-effect diode, will generate as a point source. Then the structure in question will generate the pulses of radiation, whose intensity is $\sim N^2$.

2.4. Undulator-like radiation and cooperative N^2 –effect

So far we have discussed the N^2-effect from the perspective of making undulator-like radiation coherent due to the fact that the linear size of electron bunch in the Gunn-effect diode is much smaller than the wavelength λ of the generated radiation. This point of view makes the situation very similar to the one which takes place in FEL, since the undulator periodic structure is the basic element of FEL. Let us analyse now the discussed effect from the perspective of the coherence arising in the ensemble of classical oscillators. Of course, the question is to what extent the classical model is an adequate one in this case? It is possible to demonstrate, however, that classical theory works quite well in this situation and as a matter of fact, is sufficient enough to describe it. The arguments in favour of this statement are as follows. Firstly, neither the Gunn-effect nor undulator radiation do not need quantum theory to be properly understood. Secondly, compare ε_{ph}, the energy of a single photon emitted by each of the electrons of the domain, with its mean kinetic energy ε_k. Assuming the speed of the moving strong field domain to be equal, as previously stated, $\approx 10^7$ cm s^{-1}, electron effective mass corresponding to the "heavy valley" of GaAs conduction band as to be $1.2m_e$ (m_e is free electron mass) and the radiation frequency to be $\omega \approx 3.14 \times 10^{11}$ Hz, one can easily obtain the value of ratio $\varepsilon_{ph}/\ \varepsilon_k \approx 0.006 << 1$. The last argument whatsoever indirect, appeals to an idea taken from the laser theory. It is known that there are great many approaches in a laser theory, in some of them an active medium is treated as the quantum-mechanical ensemble while the radiation as a classical electromagnetic field (see, for instance, [11]), in some others, both atoms and field, are treated quantum-mechanically [12, 13]. However, it turns out and it was shown by Borenstein and Lamb [14] that laser action can be described perfectly well in classical terms only. Hence, to our mind the process of electromagnetic field generation in the structure in question can also be considered in the frame of classical model.

2.5. The role of phasing

In this section we would like to present some evidence in favour of our statement (see *Section 2.3*) concerning the importance of the condition $l_d << a$ for N^2-effect to appear. Let us consider the system of N classical nonlinear oscillators where each one is characterized by mass m, charge e, some characteristic length a which we associate with a period of grating and the undamped angular frequency ω in the presence of an external forcing field with the angular frequency ν. Our model is similar to the one proposed in [14], yet differs from it by taking into account the radiation damping, then the evolution of the ensemble of nonlinear oscillators is described by the system of the following equations

$$\ddot{x}_i + \omega^2 x_i = f(x_i, \dot{x}_1, .., \dot{x}_N, t), \tag{9}$$

for $i = 1, \ldots, N$, where

$$f(x_i, \dot{x}_1, .., \dot{x}_N, t) = \frac{1}{6}\frac{\omega^2}{a^2} x_i^3 - \frac{2e^2\omega^2}{3m^* c^3}\sum_{j=1}^{N}\dot{x}_j + \frac{eE}{m^*}\cos(\nu t) \tag{10}$$

The first term to the right side is responsible for nonlinearity, since we do not assume the displacements to be small, they can be arbitrary. The second term represents radiation damping of electrons and the third one is nothing else, but an external driving field associated with emitted undulator-like radiation whose amplitude is E. As can be seen below, each term in the right-hand side of Eq. (9) is small compared to the terms in the left-hand. Hence, it is possible, by using the slowly varying amplitude and phase method [15], to transform the initial system of N nonlinear differential equations of the second order into the system of $2N$ nonlinear first order differential equations in the amplitude and phase variables. We are searching for a solution $x_i(t)$ to the system of Eqs. (9) in the form:

$$x_i(t) = aA_i(t)\cos(\nu t + \vartheta_i(t)), \tag{11}$$

where by hypothesis, $A_i(t)$ and $\vartheta_i(t)$ are the functions whose rate of variation is small compared with the angular frequency ν. Making some differentiations and other calculations (for details see [7]), we arrive finally at the following equations:

$$\frac{dA_i}{d\tau} = H\left[\sum_{j=1}^{N} A_j \cos(\vartheta_i(\tau) - \vartheta_j(\tau))\right] - G\sin\vartheta_i, \tag{12}$$

$$\frac{d\vartheta_i}{d\tau} = -\frac{1}{2}\frac{\nu^2 - \omega^2}{\nu^2} - \frac{1}{16}\frac{\omega^2}{\nu^2} A_i^2(\tau) - \frac{G}{A_i(\tau)}\cos\vartheta_i(\tau), \tag{13}$$

where $\tau = \nu t$ (dimensionless 'time'), $H = \frac{e^2\omega^2}{3\nu m^* c^3}$, $G = \frac{eE}{2am^*\nu^2}$. Equations (12), (13) can be solved numerically. In our calculations we set: $m^* = 1{,}2 \times 0{,}91 \times 10^{-28}$ g (effective mass of an electron in the 'heavy valley' of GaAs-conduction band), $\omega = \nu = 3{,}14 \times 10^{11}$ Hz, $E \approx 4V/cm$ = 0,0135 cgs units (assuming one hundredth of the electron energy in Gunn diode will be converted into generated radiation), a = 2×10^{-4} cm, G = 0,0015, and H = $8{,}18\times10^{-12}$. Figs. 4 and 5 present the solutions of Eqs. (12) and (13) as a series of phase diagrams where $A_i(\tau)$ is plotted as a function of $\vartheta_i(\tau)$ for the consecutive moments of dimensionless 'time' $\tau=\{0, 25, 50, 75, 100\}$. There are two cases: the first one, when initial phases of individual oscillators are uniformly distributed ranging from 0 to 2π and the second one, when initial phases are clustered around $\pi/4$ with the dispersion equal to 0.15.

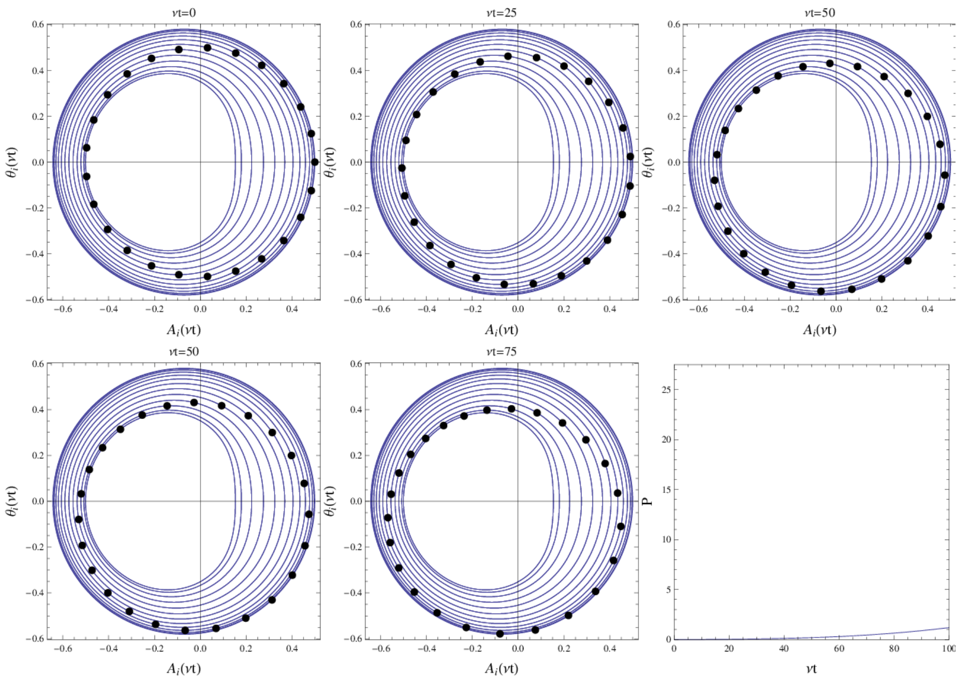

Fig 4. Phase portraits when phases of the oscillators are equally spaced in [0,2π]

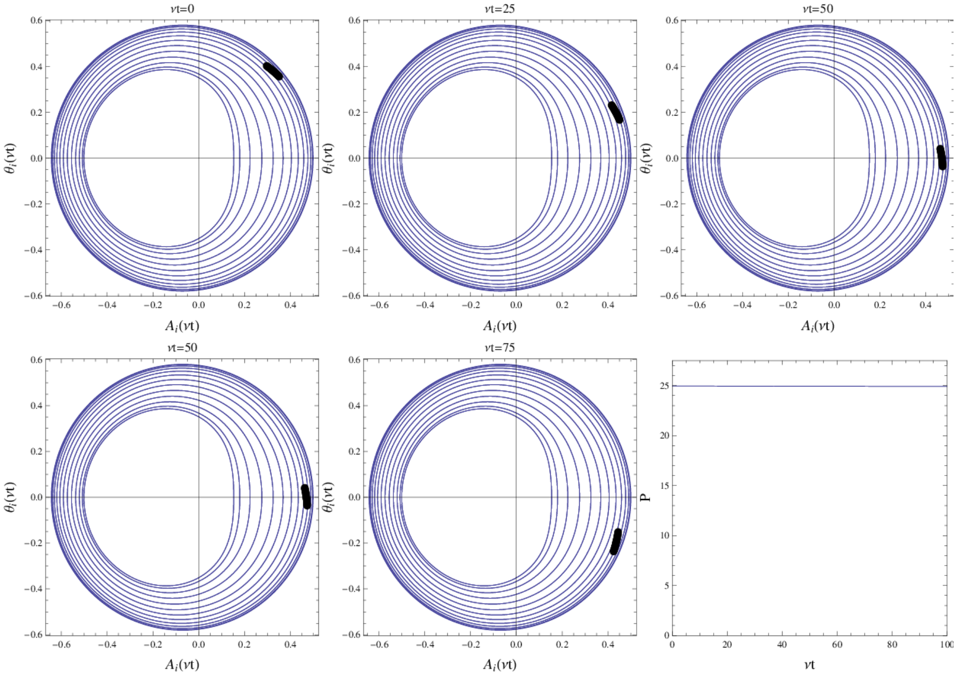

Fig 5. Phase portraits when phases of the oscillators are focused around π /4 with the dispersion equals to 0.15

Obviously, the first case corresponds to the situation when $l_d \geq a$, while the second one, when $l_d / a = 0.15$. In both cases initial amplitudes $A_i(0)$ for all oscillators are taken to be equal to 0.5. Phase diagrams are accompanied by the plot of a function $P(\tau)$ which is a measure of phasing of the electrons in a bunch, defined as:

$$P(\tau) = \frac{1}{N}\left|\sum_{j=1}^{N} \exp \vartheta_j(\tau)\right|^2 .$$

It ranges from zero when all oscillators have completely random phases, to *N* when all of them oscillate with roughly the same phase. We conclude that in the first case the driving field is too small to cause phasing of oscillators. Analyzing the second case, we set the upper bound of the sequence of moments of "time" to be 100; that corresponds to the time needed for an electron bunch to fly through the Gunn diode.

In Figs. 4 and 5 we have shown the results of calculations obtained by using the number of oscillators $N = 25$, because at the greater number of oscillators the trajectories become too dense for the phase portraits to be readable and clear. However, it should be noted that with a greater number of oscillators taken into consideration, the picture remains the same in its general features. This is because even if one assumes $N = 10^7$, the first term on the right side of Eq. (12) (the only term dependent on N), will be far smaller compared to other terms even in most favorable conditions when all phases ϑ_i coincide. It is worth noticing that based on GaAs Gunn-effect diode can generate at the frequencies up to 200 GHz, while for the proposed generator the frequencies 300 GHz and even greater ones can be achieved.

2.6. The role of form-factor

Finally, let us discuss the influence of the form of electron bunch on the N^2-effect. The total power of radiation emitted by N electrons P_N at time t is proportional to the sum of squared fields $\vec{E}(t)$ emitted by the individual electrons:

$$P_N \propto \left|\sum_{k=1}^{N} \vec{E}(t-t_k)\right|^2 = \left|\sum_{k=1}^{N} \vec{E}(t)e^{i\omega t_k}\right|^2 = P\left|\sum_{k=1}^{N} e^{i\omega t_k}\right|^2, \qquad (14)$$

where ω is the frequency of emitted radiation, P is the power generated by a single electron and t_i is time difference between the moments of time when the chosen electron, say the first one (k =1) emits its pulse and k-th one emits its own. If v *is* the electron velocity in a bunch moving along z-axis, then the term squared in (14) can be transformed in the following way:

$$\left|\sum_{k=1}^{N} e^{i\omega t_k}\right|^2 = \left|\sum_{k=1}^{N} e^{i\omega z_k/v}\right|^2 = \sum_{k=1}^{N} e^{i\omega z_k/v} \sum_{j=1}^{N} e^{-i\omega z_j/v} = N + \sum_{k=1}^{N} e^{i\omega z_k/v} \sum_{j=1,\, j\neq k}^{N} e^{-i\omega z_j/v}.$$

Assuming the electron distribution in a bunch along z-axis is characterized by the function $S(z)$ obeying the normalization condition

$$\int_{z=-\infty}^{\infty} S(z)dz = 1,$$

one can finally rewrite the formula (14) as follows (for details see [1])

$$P_N = P\left[N + N(N-1)f(z)\right], \qquad (15)$$

where

$$f(z) = \left| \int S(z) e^{i\omega z/v} dz \right|^2 ,$$

is the form-factor of electron bunch. For Gaussian distribution with mean equal to zero and standard deviation σ this form-factor is equal

$$S(z) = \frac{2}{\sqrt{2\pi\sigma^2}} e^{-\frac{z^2}{2\sigma^2}} ,$$

and the form-factor is:

$$f(z) = e^{-\left(\frac{\omega\sigma}{v}\right)^2} = \exp\left[-\left(\frac{2\pi}{1 - v/c} \right)^2 \left(\frac{\sigma}{a} \right)^2 \right], \qquad (16)$$

where a as previously means the period of grating while ω in this case is equal to

$$\omega = \frac{2\pi}{a} \frac{v}{1 - v/c} .$$

Inserting (16) into (15), one gets:

$$P_N = P\left(N + N(N-1)\exp\left(-\left(\frac{2\pi}{1 - v/c} \right)^2 \left(\frac{\sigma}{a} \right)^2 \right) \right). \qquad (17)$$

We have chosen the Gaussian distribution because most of the physical values obey this distribution. Analyzing the last formula, one can easily observe that the emitted radiation is proportional to N^2 as long as bunch form-factor is of the order of one. In the Fig. 6 we present this form-factor for different values of $\beta = v/c$.

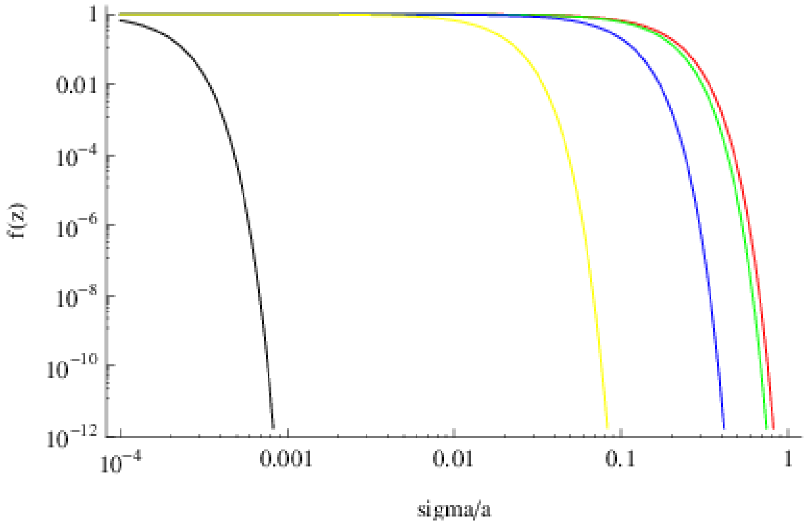

Fig. 6. The form factor of an electron bunch; line colors correspond to the different values of $\beta = v/c$ (red 0.001, green 0.05, blue 0.1, yellow 0.5)

3. THE CONCEPT OF TYPE II THZ-GENERATOR

3.1. Superradiance

The superradiance is a cooperative phenomenon, radiation occurring due to spontaneous emission in an inverted system of interacting and initially independent two-level atoms (or more generally, emitters or oscillators). It was discovered by R. Dicke long ago in his seminal work [16] and since then intensively discussed in scientific literature [17-21]. Being in superradiance state, the ensemble composed of a vast number of atoms N, which are in excited state, emits radiation during the time τ_s much smaller then τ_0, the time of spontaneous decay of excited state of an individual atom. The effect results in the radiation intensity proportional to N^2, where N is the number of emitters, and it is due to specific phase correlations arising in the ensemble of radiating atoms. In fact, the proportionality of radiation intensity to N^2 is a characteristic feature not only of the Dicke-model, since in many cases the initial correlations in the ensemble of oscillators can lead to this effect. Indeed, the possibility to get the generated power N^2 was also intensively discussed and in different contexts not in any way related to superradiance discovered by Dicke (see Section 2). That is, Dicke superradiance can occur not necessarily in the ensemble of two-level "atoms" (quantum oscillators), but in the ensemble of classical oscillators as well (see for example, the nice review [22]).These oscillators are considered as identical except the initial phases, which are or can be distributed at random. Another model which has been studied intensively in recent years, is the Kuramoto model [23]. In the model the collective behavior of the elements is considered, whose rhythmical activity is described in terms of a physical variable evolving regularly in time. Essentially the behavior of each element is similar to that of an oscillator, but the oscillators belonging to the ensemble are not necessarily identical. They can be characterized for example, by different frequencies. In the frame of this model with mean-field coupling among oscillators, the effect of phase synchronization was also discovered. It gives the possibility to look at these models from a broader perspective as a manifestation of the universal phenomenon of synchronization [24].

In the previous section 2 (initially in the paper [7]) we considered the cooperative N^2-effect which appears in the semiconductor structure with grating due to interplay and combining of two effects: undulator-like radiation and Gunn effect. The aim was to demonstrate that initial correlations in the ensemble of classical oscillators (electrons in Gun-effect diode treated in classical terms) can lead to the cooperative radiation (N^2-effect) in a structure composed of Gunn-effect diode and a properly matched grating, but not to super-radiance. The initial correlations of electrons in such structure occurs simply because the thickness of electron bunch in Gunn effect can be much smaller than the radiation wavelength generated due to undulator effect. The result is that due to combining of these two effects, the pulses of radiation occur with the frequencies of about few THz and whose power is proportional to N^2.

Thus, the purpose of this section (and the paper [25]) is to answer the following question: does the phase synchronization effect and as the consequence, the N^2effect similar to some extent to the super-radiance, occur in the ensemble of classical nonlinear oscillators with different frequencies and the phases distributed at random? Or in other words, can the effect resembling Dicke super-radiance can develop in the semiconductor structure with grating similar to that considered in Section 2, which however differs in the respect that there is no Gunn effect and hence, there is lack of initial correlations in the ensemble of classical nonlinear oscillators.

3.2. The model

Suppose we have a microstructure very similar to that described in the previous section or of that reported in [10], which was grown on semi-insulating GaAs - substrate and with the grating (electrodes arranged in a periodic sequence) on the top (Fig. 7). If the electric bias is applied to the electrodes, a weak periodic potential modulation arises within the GaAs sample and this periodic electric field becomes very similar to the periodic electric field used in some types of undulators or especially in free electron lasers (FEL), as it was already mentioned in Sec. 2. The quantum-mechanical model of a two-level system de-scribes well the spontaneous emission of atoms and molecules in electron and vibrational transitions and is widely used in quantum optics. To describe the spontaneous and induced emission in the millimeter and sub-millimeter bands it becomes necessary to use the model of classical oscillators, since the emitted photon energy in this range is low and to obtain high-power radiation the energy resource of each oscillator must exceed many times the energy of a single photon.

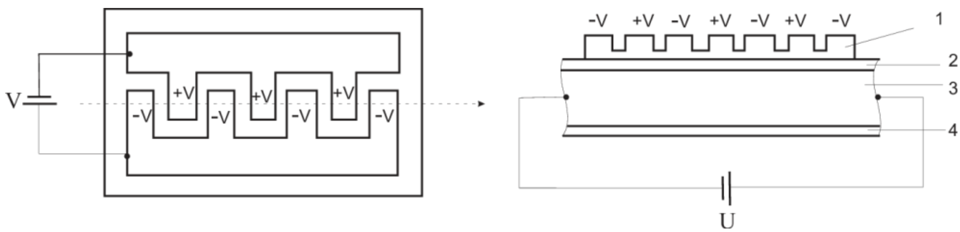

Fig. 7. Schematic diagram of generator; 1- electrode grating, 2 – insulator layer, 3 – GaAs layer, 4- substrate layer

In addition, for emission of weakly relativistic electrons in a constant field, such as for instance, in cyclotron-resonance masers, the oscillating-electron anharmonicity due to the relativistic correction to the Hamiltonian can be much less than the constant of the interaction with the radiation field. Therefore, many levels participate in the emission and absorption processes even in the resonance approximation, and the two-level approximation becomes irrelevant. Instead, the model of nonlinear classical oscillators seems to be most appropriate to describe the situation, that is why in previous section as well as in this one namely this model is used.

Let us consider the system of N classical nonlinear oscillators when each of them is characterized by mass m, charge e, some characteristic length having the physical meaning of the amplitude and which we associate with a period of grating a and the undamped angular frequency ω_i in the presence of an external forcing field with the angular frequency ν. We suppose ν to be equal to $\langle \omega_i \rangle = \omega_0 = a / v_d$, where v_d is the electron drift velocity. We assume also, that ω_i, the frequencies of the individual oscillators are distributed around ω_0 with the deviations not greater than ten percent.

Then the evolution of the ensemble of nonlinear charged oscillators is described by the system of equations analogous to the set of equations (9), but with the right-hand side different, namely:

$$f(x_i, \dot{x}_1, .., \dot{x}_N, t) = \frac{1}{6}\frac{\omega_i^2}{a^2}x_i^3 - \frac{2e^2\omega_i^2}{3m^* c^3}\sum_{j=1}^{N}\dot{x}_j + \frac{e}{m^*}\left(E_0 + E(t)\right)\cos(\nu t). \quad (18)$$

The first two terms in (18) differ from those ones in (9) in the respect that now we do not assume the oscillation frequencies of individual electrons are equal, quite the contrary they are different. The third term is nothing else but an external driving field associated with the 'pumping wave' related to the electron motion within the undulator (the term E_0) and the backward effect of the emitted radiation (the term ($E(t)$).

Again, we use the slowly varying amplitude and phase method [15], to transform the initial system of N nonlinear differential equations of the second order into the system of $2N$ nonlinear first order differential equations in the amplitude and phase variables. Finally, we reduce this set of equations to the following one:

$$\frac{dA_i}{d\tau} = H_i \left[\sum_{j=1}^{N} A_j \cos(\vartheta_i(\tau) - \vartheta_j(\tau)) \right] - G \sin \vartheta_i , \tag{19}$$

$$\frac{d\vartheta_i}{d\tau} = -\frac{1}{2}\frac{\nu^2 - \omega_i^2}{\nu^2} - \frac{1}{16}\frac{\omega_i^2}{\nu^2} A_i^2(\tau) - \frac{G}{A_i(\tau)} \cos \vartheta_i(\tau) , \tag{20}$$

where $H_i = \dfrac{e^2 \omega_i^2}{3\nu mc^3}$, $G = \dfrac{e(E_0 + E(P))}{2am\nu^2}$, and P is the measure of phasing (see below).

3.3. The results of numerical calculations

Eqsuations (19) and (20) can be solved numerically. In our calculations we set m=0.067m_e, effective mass of an electron in the central valley of GaAs-conduction band, where m_e is the free electron mass, v_d = 5×10^{5}cm/s, ν =1.57×10^{10}Hz, E_0≈0.1V/cm=0.0003375cgs units, a=2×10−4cm.

In the simulations we assume random distribution of initial amplitudes as well as the frequencies of the individual oscillators ω_i, and their phases. The frequencies are considered as distributed around $\nu=\omega_0$ at random within the range of 10 percent deviation (dispersion), while the initial phases were also distributed at random ranging from 0 to 2π. As for the field $E(P)$ which is generated by the electrons moving in a periodic field, we assume that $E(P) = E_{min}+(E_{max}-E_{min})P(\tau)$, where $P(\tau)$, is a measure of phasing of the electrons in a bunch, now defined as:

$$P(\tau) = \frac{1}{N^2} \left| \sum_{j=1}^{N} \exp \vartheta_j(\tau) \right|^2 .$$

It ranges from zero when all oscillators have completely random phases, to one when all of them oscillate with roughly the same phase. It is worth mentioning that the measure introduced by us for the purposes of this study, although resembles the measure of ‘order parameter’ introduced in discussing the Kuramoto model [23], differs however from it. The former is real number, while the ‘order parameter’ of Kuramoto model is complex- valued.

As for the values of $E_{\min}$ and $E_{\max}$, they were calculated by equating the radiation power emitted by N classical dipoles with the absolute value of Poynting vector times area. This leads to the next expressions:

$$E_{\min} \sim \frac{4\sqrt{\pi}\, ea\omega_0^2 N^{1/2}}{c^2\sqrt{S}}\ ,\ E_{\max} \sim \frac{4\sqrt{\pi}\, ea\omega_0^2 N}{c^2\sqrt{S}}\ ,$$

where S is the area of the semiconductor layer. We assume the number of the periods of grating to be equal to 30, which means that the total length of the structure becomes equal to 6×10^{-3} cm and the 'dimensionless ‘time’ τ (the time electrons needed to cover this distance) is about 200.

As it is seen in Figs. 8(a) and 8(b), the phasing in the ensemble of classical nonlinear oscillators indeed at first develops and reaches almost 100 percent and then after some time disappears. Since we treat this model as describing the electrons moving in the periodic field of the ‘micro-undulator,’ we can conclude that combined action of periodic undulator field and the backward effect of radiation emitted by the electrons can lead to the N^2-effect. Thus, we answer the question posed in the beginning of this section in affirmative. Comparing the results obtained in Section 2 we conclude that there are some differences between them. First, despite the fact that there are no initial correlations in the ensemble of electrons, the phasing appears. However, while in the former case, (that is, in case of combining Gunn effect and undulator field) the phasing exists during the time electrons need to cover the distance equal to the length of the device, in the last case the phasing disappears after some time. This seems intuitively clear: after emitting the radiation pulse whose intensity is $\sim N^2$, the oscillators should have some time for the rearrangement of their phases. It is worth mentioning that since in each individual simulation we choose initial amplitudes, frequencies and phases of the oscillators at random, the phasing function each time has a slightly different form as it can be seen in Fig. 8(a), the upper frame. It is worth emphasizing also that because we assumed the total randomness of initial phases of the individual oscillators, sometimes in the individual simulation the phase changes could not be considered as slow with respect to $1/\omega_0$. It means that sometimes the accepted preconditions that $A_i(t)$ and $\theta_i(t)$ are the functions whose rate of variation is small compared with the angular frequency ν are violated and one should be very careful in doing such simulations excluding these situations.

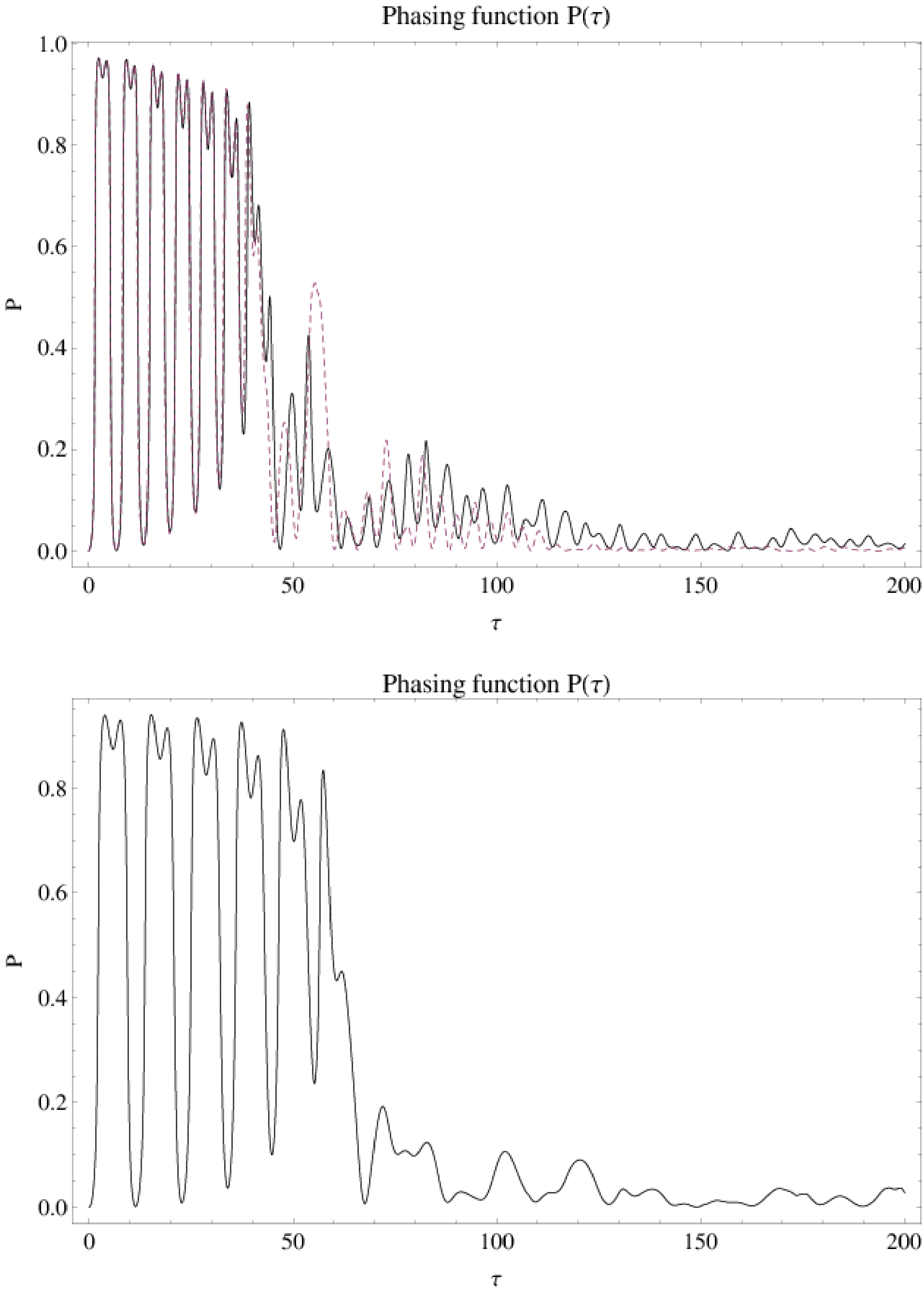

Fig. 8. (a) - the upper panel: time evolution of the $P(\tau)$ function; two lines, solid and dashed, correspond to two different amplitude distributions, phases and frequencies at the initial time, the electron concentration is assumed to be 10^{17} cm^{-3}; (b) – the lower panel: time evolution of the $P(\tau)$ function; the electron concentration is assumed to be 10^{16} cm^{-3}

4. CONCLUSION

In this paper we have considered cooperative N^2-effect which to our mind, can occur in the GaAs-structure with grating. In the first model studied by us the effect is the result of interplay and combining two other effects, namely Gunn-effect and the undulator-like radiation. The mechanism which leads to the proportionality of radiation power to N^2 is the initial phasing of electrons in a bunch due to Gunn effect and the fact that the linear size of a bunch l_d is much smaller than the period of grating a ($l_d \ll a$) as well as the radiation wavelength. Treating the second model, we have considered the time evolution of a great number of coupled nonlinear charged oscillators which interact with the external driving electric field and with each other by means of radiation field. This model corresponds to the electrons moving in the periodic electric field of 'micro-undulator' composed of the GaAs-semiconductor layer with the properly matched grating on the top but with the lack of initial phasing that is, in the absence of Gunn effect. The external periodic driving field is nothing else but the undulator field and it is analogous for instance, to the 'pumping wave' in maser or laser. In the frame of the model, we also took into account the backward effect of the radiation field on the electrons moving within the micro-undulator. The effect resembles to some extent the Dicke superradiance but differs from it in the respect that the oscillators not only interact with each other via the emitted radiation but rather are under the influence of the undulator field. On the other hand, it also resembles the synchronization occurring in the Kuramoto model. This one enables to put it in a broader context, namely the universal phenomenon of synchronisation. In the simulations we assumed the frequencies of the oscillators to be distributed at random around some definite frequency and the randomness of the initial phases of oscillators. It seems very probable that the predicted effects can be used for the developing of generators which could produce radiation at the frequencies even up to a few THz at room temperature.

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